4 Response of an Unlined Circular Tunnel in a Biaxial Stress Field

4.1 Problem Statement

Crushing failure is an important mechanism by which unlined tunnels may fail. Crushing is treated as a static phenomenon and involves massive failure around the excavation due to large-scale plastic flow. The purpose of this verification example is to demonstrate the ability of *UDEC* to model large-scale plastic flow.* The verification was accomplished by comparing *UDEC* results to those from a closed-form solution that includes plastic flow behavior.

The problem involves a circular tunnel subjected to a non-hydrostatic static load. The medium surrounding the tunnel is treated as an elasto-plastic material with failure defined by a Mohr-Coulomb yield function. The dilatancy of the material at failure is defined by the flow rule, which is characterized by the dilatancy angle. Both fully dilatant and non-dilatant material behaviors are verified.

The objective of this problem is to test the elasto-plastic material model used to describe the nonlinear deformational behavior of fully deformable blocks in *UDEC*. This test specifically addresses the ability of the code to simulate plastic flow accurately.

4.2 Analytical Solution

Two conventional closed-form techniques used for preliminary analyses of circular tunnels subjected to far-field mechanical loading are the solutions presented by Newmark et al. (1970) and Hendron and Aiyer (1971). These solutions idealize the problem as a static, two-dimensional analysis of a circular tunnel in a hydrostatic stress field. The surrounding medium is treated as an elasto-plastic material with failure defined by a Mohr-Coulomb yield function. The dilatancy of the material at failure is defined by the plasticity flow rule, which is characterized by the dilatancy angle. The Newmark solution assumes a fully nonassociated flow rule (i.e., no dilatancy occurs at failure). The Hendron and Aiyer solution assumes a fully associated flow rule (i.e., the dilatancy angle equals the friction angle).

Detournay (1983) obtained the general solution for non-hydrostatic loading by the development of a semi-analytical technique. This approach applies for arbitrary dilatancy of the material, and therefore makes the solutions of Newmark and Hendron and Aiyer special cases of the Detournay solution. For this reason, the Detournay solution was selected as a more rigorous verification test of *UDEC*.

Note that all three solutions are based on infinitesimal (small) strain theory, which assumes that the initial geometry of a deforming body is not appreciably altered during the deformation process. The consequence of this assumption is discussed later.

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The normalized stresses and displacements of the problem can be written in dimensionless form, as functions of independent variables and problem parameters:

$$\frac{\sigma_{ij}}{q} = \sigma^*_{ij}\left[\frac{r}{a},\ \theta,\ \frac{(\sigma_1+\sigma_2)}{q},\ \frac{(\sigma_1-\sigma_2)}{q},\ \phi,\ \psi\right] \tag{4.1}$$

$$2\frac{u_r}{a}\frac{G}{q} = U_r\left[\frac{r}{a},\ \theta,\ \frac{(\sigma_1+\sigma_2)}{q},\ \frac{(\sigma_1-\sigma_2)}{q},\ \nu,\ \phi,\ \psi\right] \tag{4.2}$$

where σ_{ij} = stresses;
u_r = radial displacement;
r = radial coordinate;
θ = angle;
a = tunnel radius;
q = uniaxial compressive strength;
σ_1, σ_2 = far-field principal stresses;
ν = Poisson's ratio;
G = shear modulus;
ϕ = internal friction angle; and
ψ = dilation angle.

Therefore, the normalized radial displacement of crown (i.e., $r/a = 1,\ \theta = \pi/2$) of the tunnel excavated in the rock characterized by given friction angle, ϕ_o, dilation angle, ψ_o, and Poisson's ratio, ν_o, can be written as

$$U_r^c = U_r\left[1,\ \pi/2,\ \frac{(\sigma_1+\sigma_2)}{q},\ \frac{(\sigma_1-\sigma_2)}{q},\ \nu_o,\ \phi_o,\ \psi_o\right] \tag{4.3}$$

while the normalized radial displacement of springline ($r/a = 1,\ \theta = 0$) is

$$U_r^s = U_r\left[1,\ 0,\ \frac{(\sigma_1+\sigma_2)}{q},\ \frac{(\sigma_1-\sigma_2)}{q},\ \nu_o,\ \phi_o,\ \psi_o\right] \tag{4.4}$$

Functions U_r^c and U_r^s can be conveniently represented in the form of design charts. Figure 4.1 presents two charts: (1) for associative material, $\phi_o = 30°$ and $\psi_o = 30°$; and (2) for non-associative material, $\phi_o = 30°$ and $\psi_o = 30°$ (in both cases, Poisson's ratio, ν_o = 0.25). The normalized displacements due to tunnel excavation for any free-field state of stress can be read from the charts by interpolating between the plotted contours. Actual radial displacements, u_r, can be calculated from the normalized displacements, U_r, from

$$u_r = \frac{a\,q}{2G}\,U_r \tag{4.5}$$

Alternatively, the percentage closure $\hat{u}_r = 100\,(u_r/a)$ can be expressed as

$$\hat{u}_r = \frac{50q}{G}\,U_r \;\;(\%) \tag{4.6}$$

These displacements apply for the case of a tunnel excavation in a rock mass previously stressed to the far-field stress state. The charts, therefore, calculate displacements due to the initial state of stress. The displacements induced by additional external loading differ from those calculated by the charts by an amount equal to the elastic displacements that would occur in the absence of the tunnel. The corrections for added external loading are

at the crown

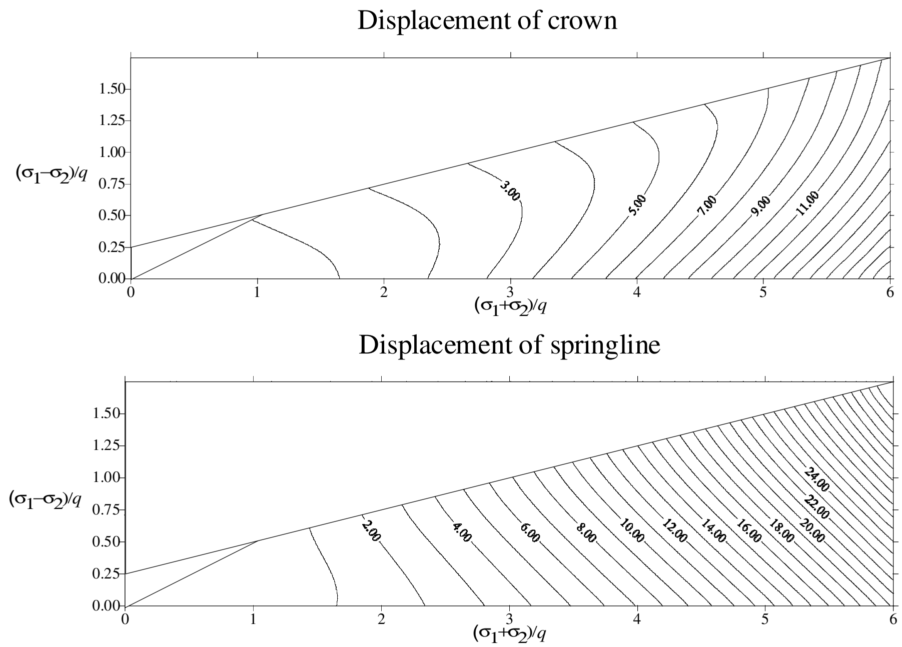
$$\Delta U_r^c = (1-2\nu)\,\frac{\sigma_1+\sigma_3}{2q} + \frac{\sigma_1-\sigma_3}{2q} \tag{4.7}$$

at the springline

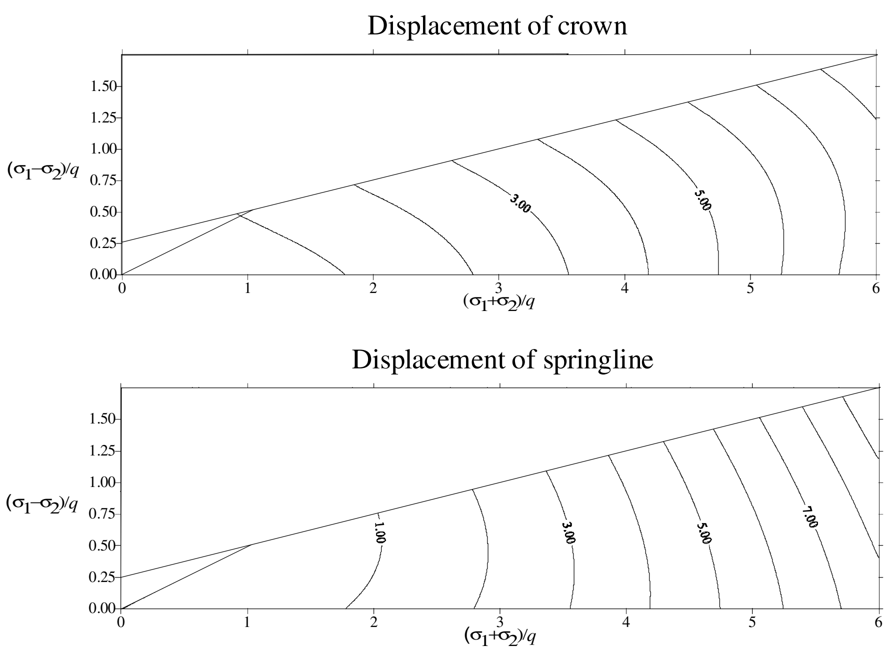
$$\Delta U_r^s = (1-2\nu)\,\frac{\sigma_1+\sigma_3}{2q} - \frac{\sigma_1-\sigma_3}{2q} \tag{4.8}$$

The percentage closure for added external loading is then

$$u_r^k = \frac{50q}{G}\,(U_r^k + \Delta U_r^k) \quad k = c,\; s \tag{4.9}$$



(a) fully dilatant ($\phi = \psi = 30°$)



(b) no dilatancy ($\phi = 30°$; $\psi = 0$)

Figure 4.1 Normalized radial displacements (U_r) results of closed-form solution

4.3 *UDEC* Model

The *UDEC* model is automatically generated (using *FISH*) as a function of independent dimensionless parameters of the problem (i.e., ν, ϕ, ψ and normalized far-field stresses) and normalized lengths pertinent to the discrete numerical model (i.e., normalized zone size, ℓ_z/a, and normalized distance to the far-field boundaries, ℓ_M/a).

Two particular cases have been solved in this verification problem:

Material Properties	Case I	Case II
Poisson's ratio (ν)	0.25	0.25
angle of internal friction (ϕ)	30°	30°
dilation angle (ψ)	30°	0

Far-field stresses that satisfy the relation

$$\frac{(\sigma_1 - \sigma_2)}{q} = 0.25 \frac{(\sigma_1 + \sigma_2)}{q}$$

are applied in steps (expressed in normalized form), as shown in Table 4.1.

Table 4.1 Loading steps for circular tunnel in a non-hydrostatic stress field

Step	$\frac{\sigma_1+\sigma_3}{q}$	$\frac{\sigma_1-\sigma_3}{q}$
	1.0	0.25
1	1.5	0.375
2	2.0	0.5
3	2.5	0.625
4	3.0	0.75
5	3.5	0.875
6	4.0	1.0

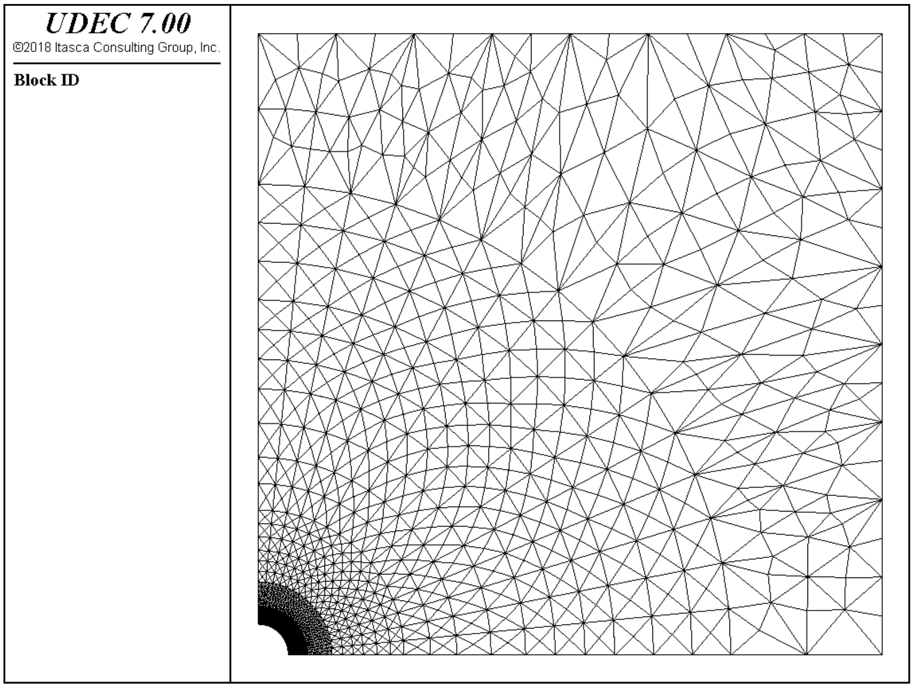
The *UDEC* model for the given test problem is illustrated in Figure 4.2. The model is one quadrant of the tunnel and surrounding rock. The bottom and left boundaries shown in the figure are lines of symmetry. The model is divided into a series of concentric "rings" with increasing spacing between "ring" cuts. In this way, the block zoning can be increased away from the hole. In the first few "rings" adjacent to the hole, it is possible to create a mesh of diagonally opposed triangular zones. The zoning, generated from the condition that $\ell_z/a = 0.05$ in the first "ring" around the tunnel, is shown in Figure 4.3.

Figure 4.2 Plot of "glued" joints in UDEC model used to improve zonal discretization in model

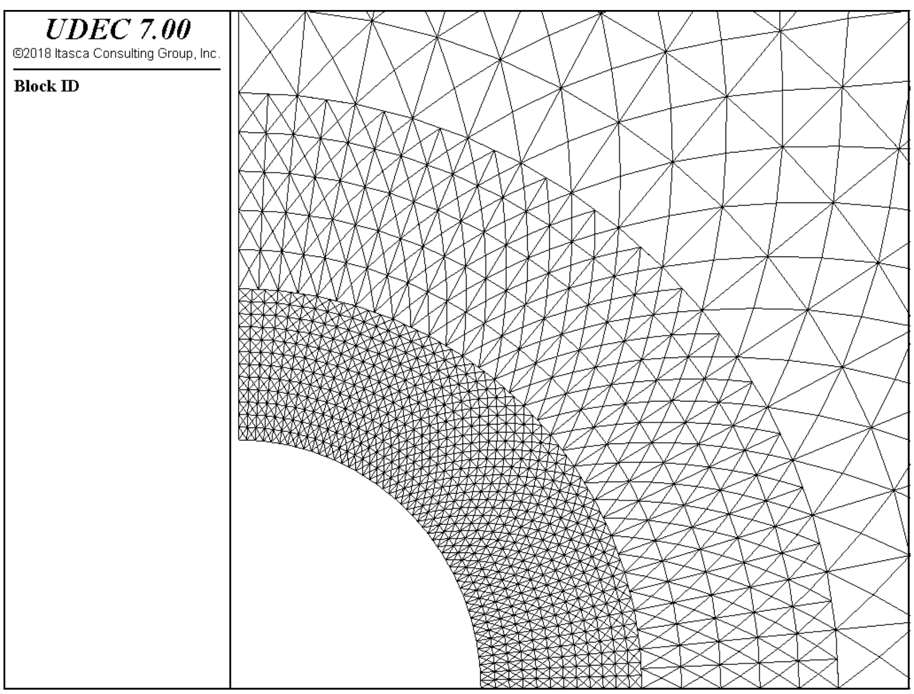
Particular dimensional variables in these simulations are

tunnel radius (a)	0.254 m
shear modulus (G)	4.49 GPa
cohesion (c)*	9.95 MPa

* Unconfined compressive strength is related to cohesion by $q = \frac{2c\cos\phi}{1-\sin\phi}$.



(a) problem discretization



(b) problem discretization near tunnel periphery

Figure 4.3 UDEC model

The tunnel closure results are very sensitive to the location of the model boundaries. Goodman (1980, p. 236) notes that plastic behavior of the region in the vicinity of a tunnel has the effect of extending the influence of the tunnel a considerable distance into the surrounding rock. For elasto-plastic behavior, a distance of 10 tunnel radii from the tunnel is required to bring the stress perturbation to within 10% of the initial stress state. Therefore, the model is generated such that $\ell_M/a = 20$.

The joints between the blocks are "glued" by setting the cohesion and tensile strength of the contacts to values much higher than the applied loads. The normal and shear stiffnesses of the joints are set equal to a value that produces an equivalent elastic modulus for the model within 1.5% of the given Young's modulus.

4.4 Results and Discussion

Analytically calculated radial displacements of the springline and crown for the problem parameters used in numerical simulations are summarized in Table 4.2. These results demonstrate the significant influence of dilatancy on the deformation of the tunnel at the springline. For these problem conditions, the closure at the springline is nearly three times greater for the dilatant material versus the non-dilatant material, while the closure at the crown is virtually unaffected.

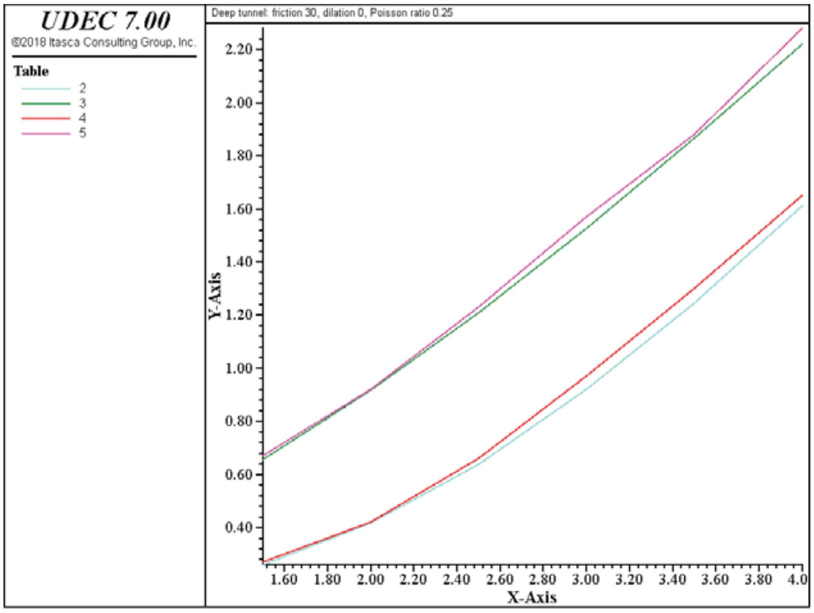
Table 4.2 Calculated closure from Detournay solution

	$\frac{\sigma_1-\sigma_3}{q}$	$\frac{\sigma_1+\sigma_3}{q}$	U_r^s	ΔU_r^s	$\hat{u}_r^s$ %	U_r^c	ΔU_r^c	$\hat{u}_r^c$ %
$\psi = 0$	0.375	1.5	0.55	0.19	0.27	1.25	0.56	0.67
	0.50	2.0	0.9	0.25	0.42	1.75	0.75	0.92
	0.625	2.5	1.5	0.31	0.66	2.4	0.94	1.23
	0.75	3.0	2.25	0.38	0.97	3.15	1.12	1.57
	0.875	3.5	3.1	0.44	1.30	3.8	1.31	1.88
	1.0	4.0	4.0	0.5	1.65	4.7	1.5	2.28
$\psi = 30°$	0.375	1.5	0.55	0.19	0.27	1.25	0.56	0.67
	0.50	2.0	2.0	0.25	0.83	1.75	0.75	0.92
	0.625	2.5	3.75	0.31	1.49	2.35	0.94	1.21
	0.75	3.0	6.0	0.38	2.35	3.05	1.12	1.53
	0.875	3.5	9.0	0.44	3.47	3.8	1.31	1.88
	1.0	4.0	12.8	0.50	4.89	4.75	1.5	2.30

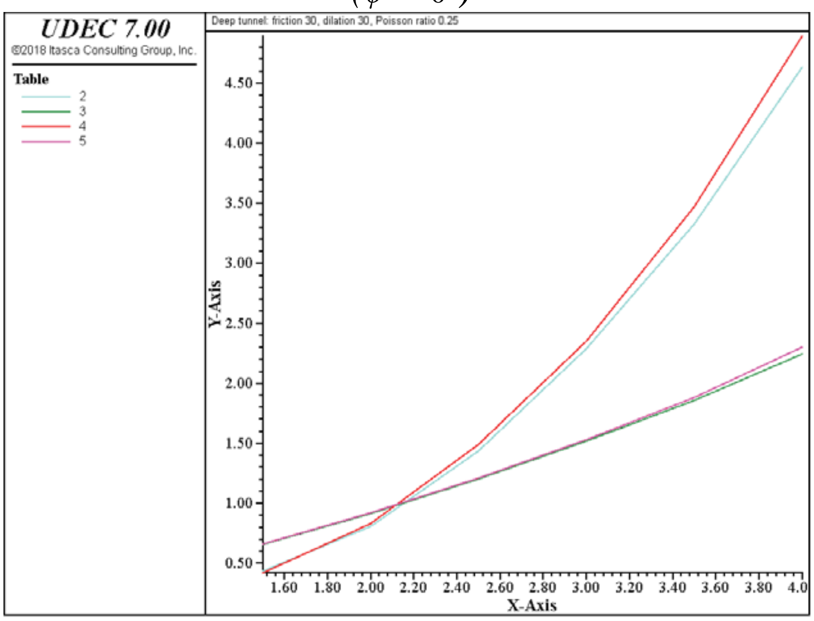
The comparison of the *UDEC* results to the Detournay solution is given in Table 4.3 and, graphically, in Figure 4.4.

Table 4.3 Comparison of UDEC results to Detournay solution

	Crown Closure $\hat{u}_r^c$			Springline Closure $\hat{u}_r^s$		
	Analytic Solution (%)	*UDEC* %	Error %	Analytic Solution (%)	*UDEC* %	Error %
Elastic	0.620	0.619	−0.2	.207	0.203	−2.0
Elasto-Plastic ($\psi = 0°$)						
Step 1	0.67	0.66	−1.5	0.27	0.26	−3.7
2	0.92	0.92	0.0	0.42	0.42	0.0
3	1.23	1.21	−1.6	0.66	0.64	−3.0
4	1.57	1.53	−2.5	0.97	0.92	−5.2
5	1.88	1.87	−0.5	1.30	1.24	−4.6
6	2.28	2.22	−2.6	1.65	1.61	−2.4
Elasto-Plastic ($\psi = 30°$)						
Step 1	0.66	0.66	0.0	0.42	0.44	4.8
2	0.92	0.91	−1.1	0.83	0.81	−2.4
3	1.21	1.20	−0.8	1.49	1.44	−3.4
4	1.53	1.52	−0.7	2.35	2.29	−2.6
5	1.88	1.86	−1.1	3.47	3.33	−4.0
6	2.30	2.24	−2.6	4.89	4.63	−5.3



($\psi = 0°$)



($\psi = 30°$)

Figure 4.4 ***Comparison of crown and springline closures for UDEC and analytic solutions***
Table 2 – U_r^s, UDEC
Table 3 – U_r^c, UDEC
Table 4 – U_r^s, analytic solution
Table 5 – U_r^c, analytic solution

Constant-strain triangular elements such as those used in *UDEC* tend to inhibit incompressible plastic flow, and may produce an excessively stiff and incorrect calculation for plastic flow. Nagtegaal et al. (1974) discuss procedures to improve the representation of plastic flow for triangular elements. One technique is the mixed-discretization procedure (Marti and Cundall 1982), which reduces the constraints on plastic flow by using different numerical discretization for the isotropic and deviatoric parts of the strain tensor. This scheme works well for uniform grids composed of equal pairs of triangular elements. Mixed-discretization is not used in *UDEC* because the creation of arbitrarily shaped blocks makes the discretization of uniform grids of paired triangular elements difficult. An alternative approach, used in *UDEC* for this test problem, is to first divide the model such that a grid of diagonally opposed triangles can be generated immediately adjacent to the excavation. Nagtegaal et al. (1974) show that meshes composed of diagonally opposed triangles also will produce a good representation for plastic flow.

The material deformation model used in *UDEC* is based upon finite strain theory. Comparisons between small- and large-strain calculations made by others (e.g., Carter et al. 1977) demonstrate that at a given strain level, compressive stresses will be higher for a large-strain calculation than for a small-strain calculation. This difference is attributed to the change in stress-rate vector as well as the change in strain-rate vector, which is accounted for in the large-strain formulation and leads to increased stress concentration with increased deformation. Hence, the small-strain formulation used in the closed-form solutions will give a more conservative (higher) calculation for tunnel closure than that calculated with the large-strain formulation.

The large closure produced for the given problem conditions, particularly at the associated flow state, poses a rigorous test for the failure model used in *UDEC*. Problems that involve large strain and collapse require a numerical scheme that allows locally incompressible plastic flow.

The plasticity model appears to perform correctly in *UDEC*. In general, the agreement with the Detournay solution is reasonable; the average error can be attributed to the differences between the small- and large-strain formulation. However, a fine mesh and model boundaries at least 10 tunnel radii from the tunnel are required to produce accurate displacement calculations for plasticity analysis with the code.

4.5 References

Carter, J. P., J. R. Booker and E. H. Davis. "Finite Deformation of an Elasto-Plastic Soil," *Int. J. Num. & Analy. Meth. Geomech.*, ***1***, 25-43 (1977).

Detournay, E. "Two-Dimensional Elasto-Plastic Analysis of a Deep Cylindrical Tunnel Under Non-Hydrostatic Loading." Ph.D. Dissertation, University of Minnesota (1983).

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Hendron, A. J., Jr., and A. K. Aiyer. "Stresses and Strains Around a Cylindrical Tunnel in an Elasto-Plastic Material with Dilatancy," U.S. Army Corps of Engineers, Omaha District (January 1971).

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Nagtegaal, J. C., D. M. Parks and J. R. Rice. "On Numerically Accurate Finite Element Solutions in the Fully Plastic Range," *Comp. Meth. Appl. Mech.*, ***4***, 153-177 (1974).

Newmark, N. M., et al. "Ground Motion Technology Review," Nathan M. Newmark Consulting Engineering Services (Urbana, Illinois), SAMSO, TR-70-114 (April 1970).

4.6 Listing of Data Files

Example 4.1 TUN_IN.DAT

```
new
;----------------------------------------------------------
; Verification test:
; Circular tunnel in non-hydrostatic stress field
;
; Input data
;----------------------------------------------------------
;
; case 1 - associative material
;
;----------------------------------------------------------
call 'cont_tun.fis'
;
; --- friction angle ---
;
fish set @phi 30.
;
; --- dilation angle ---
;
fish set @ksi 30.
;
; --- Poisson's ratio ---
;
fish set @poisson 0.25
;
; --- loading path ---
;
table 1 add 1.5 0.375 2.0 0.5 2.5 0.625 3.0 0.75 3.5 0.875 4.0 1.0
;
; --- zone size ---
;
fish set @a_z 20.
;
; --- model size ---
;
fish set @r_a 20.
;
; --- analytic solution for comparison ---
; --- springline ---
;
table 4 add 1.5 0.42 2.0 0.83 2.5 1.49 3.0 2.35 3.5 3.47 4.0 4.89
;
```

```
; --- crown ---
;
table 5 add 1.5 0.66 2.0 0.92 2.5 1.21 3.0 1.53 3.5 1.88 4.0 2.30
call 'tun.dat'
new
;---------------------------------------------------------
;
; case 2 - non-associative material
;
;---------------------------------------------------------
call 'cont_tun.fis'
fish set @phi 30.
fish set @ksi 0.
fish set @poisson 0.25
table 1 add 1.5 0.375 2.0 0.5 2.5 0.625 3.0 0.75 3.5 0.875 4.0 1.0
fish set @a_z 20.
fish set @r_a 20.
table 4 add 1.5 0.27 2.0 0.42 2.5 0.66 3.0 0.97 3.5 1.30 4.0 1.65
table 5 add 1.5 0.67 2.0 0.92 2.5 1.23 3.0 1.57 3.5 1.88 4.0 2.28
call 'tun.dat'
```

Example 4.2 CONT_TUN.FIS

```
;-----------------------------------------------------------
; Verification test:
; Circular tunnel in non-hydrostatic stress field
;
; Model preparation, control and post-processing
;-----------------------------------------------------------
;
fish define setup
;
; --- title of simulation ---
;
  ta      = int(phi)
  pa      = int(ksi)
  psn     = int(100.*poisson)
  run_t   = 'Deep tunnel: friction '+string(ta)+', dilation '+string(pa)
  run_t   = run_t+', Poisson ratio 0.'+string(psn)
  nam_t   = 'tf'+string(ta)+'d'+string(pa)
  nam_geo = nam_t+'g.sav'
  nam_e = nam_t+'e.sav'
;
; --- UDEC parameters ---
;
  a = 0.254
  coh_v = 9.95e6
  shear_m = 4.69e9
  bulk_m = 2.*(1.+poisson)*shear_m/(3.*(1.-2.*poisson))
  ucs = 2.0*coh_v*math.cos(phi*math.degrad)/(1.-math.sin(phi*math.degrad))
;
; --- parameters used in generation of model geometry ---
;
  large = bulk_m*1e10
  if a_z < 5. then
    a_z = 5.
  end_if
  if r_a < 5. then
    r_a = 5.
  end_if
  zone = a/a_z
  j_stiff = 10.*(bulk_m+4.*shear_m/3.)/zone
  mod_size = a*r_a
  ahalf = 0.5*a
  small_len = 0.2*zone
  n_small_len = -small_len
```

```
  a_l = a - small_len
  a_u = a + small_len
  mod_size_u = 1.02 * mod_size
  mod_size_l = 0.98 * mod_size
  rrat   = 1.3
end
;
fish define rings
;
; --- generation of circular rings ---
;
  d1 = 0.5*a_z*zone
  if rrat # 1. then
    nrings = math.ln(1.-(mod_size-a)*(1.-rrat)/d1)/math.ln(rrat)
    nrings = int(nrings)
    coef = nrings*math.ln(rrat)
    coef = math.exp(coef)
    d1 = (mod_size-a)*(1.-rrat)/(1.-coef)
  else
    nrings = int((mod_size-a)/d1)
    d1 = (mod_size-a)/nrings
  end_if
  di = d1
  dr = a
  loop i (1,nrings)
    command
      block cut tunnel 0 0 @dr 64
    end_command
    dr = dr + di
    di = di*rrat
  end_loop
end
;
fish define gzones
;
; --- zoning of the model ---
;
  dr  = a
  di  = d1
  dri = dr + di
  jump = 2
  loop i (1,nrings-1)
    command
      block zone generate  quad @zone ...
        range annulus center 0 0 radius @dr @dri
    end_command
```

```
      zone = jump*dri*zone/dr
      if i > 2 then
        jump = 1
      end_if
      dr = dri
      di = di * rrat
      dri = dri + di
      if zone > 0.5*(dri-dr) then
        zone = 0.5*(dri-dr)
      end_if
    end_loop
    zone = 0.5*(mod_size - dri + di)
    if zone < 0.1*mod_size then
      zone = 0.1*mod_size
    end_if
    if zone > 0.25*mod_size then
      zone = 0.25*mod_size
    end_if
    command
      block zone generate  edge @zone
    end_command
  end
  ;
  fish define ff_load
  ;
  ; --- boundary conditions from table 1 and cycling of the UDEC
  ;     model
  ;
    crown  = block.gp.near(0,a)
    spring = block.gp.near(a,0)
    ff_v   = block.gp.near(0,mod_size)
    ff_h   = block.gp.near(mod_size,0)
    i = 1
    dev = table.y(1,i)
    sph = table.x(1,i)
    loop while table.x(1,i) # 0
      syy = -0.5*ucs*(dev+sph)
      sxx = -0.5*ucs*(sph-dev)
      command
        block mechanical damp initial
        block edge apply stress 0 0 @syy ...
          range pos-x @n_small_len @mod_size_u pos-y @mod_size_l @mod_size_u
        block edge apply stress @sxx 0 0 ...
          range pos-x @mod_size_l @mod_size_u pos-y @n_small_len @mod_size_U
        block gridpoint apply velocity-x 0.0 ...
          range pos-x @n_small_len @small_len pos-y @n_small_len @mod_size_u
```

```
      block gridpoint apply velocity-y 0.0 ...
        range pos-x @n_small_len @mod_size_u pos-y @n_small_len @small_len
      block solve ratio 2e-6
    end_command
    if i = 1 then
      command
        model save @nam_e
; --- allow plastic failure ---
        block change model 3
        block property material 1 cohesion @coh_v
        block solve ratio 2e-6
      end_command
    end_if
    nam_p = nam_t+string(i)+'.sav'
    table.x(2,i) = table.x(1,i)
    table.y(2,i) = 100.*math.abs(block.gp.disp.x(spring))/a
    table.x(3,i) = table.x(1,i)
    table.y(3,i) = 100.*math.abs(block.gp.disp.y(crown))/a
    command
      model save @nam_p
    end_command
    if math.abs(block.gp.pos.x(ff_h)) > math.abs(block.gp.pos.y(ff_v)) then
      mod_size_l = 0.98*block.gp.pos.y(ff_v)
    else
      mod_size_l = 0.98*block.gp.pos.x(ff_h)
    end_if
    i = i+1
    sph = table.x(1,i)-table.x(1,i-1)
    dev = table.y(1,i)-table.y(1,i-1)
  end_loop
end
return
```

Example 4.3   TUN.DAT

```
;---------------------------------------------------------
; Verification test:
; Circular tunnel in non-hydrostatic stress field
;
; UDEC commands
;---------------------------------------------------------
;
; --- problem setup ---
;
@setup
```

```
model title @run_t
round = 0.002
;
; --- generation of geometry
;
block 0 0 0 @mod_size @mod_size @mod_size @mod_size 0
@rings
block delete range pos-x 0 @ahalf pos-y 0 @ahalf
@gzones
model save @nam_geo
;
; --- define material properties ---
;
prop mat=1 den=1 bulk =@bulk_m shear=@shear_m
;
; --- model is cycled elastically for the first loading step
;
prop mat=1 coh=@large fri=@phi dil=@ksi ten=@large
;
; --- glue joints ---
;
prop mat=1 jkn=@j_stiff jks=@j_stiff jcoh=@large jten=@large
;
set dscan 10000
;
; --- local damping ---
;
damp local
insitu stress -1 0 -1 szz -0.5
;
; --- histories ---
;
hist interval 50
hist xdis (@a,0)
hist ydis (0,@a)
hist damp
model display hist 1
;
@ff_load
ret
```