8 Rough Footing on a Mohr-Coulomb Material

8.1 Problem Statement

The prediction of collapse loads under steady plastic flow conditions is one that can be difficult for a numerical model to simulate accurately (Sloan and Randolph 1982). A simple example of a problem involving steady flow is the determination of the bearing capacity of a footing on an elastic-plastic soil. The bearing capacity is dependent on the steady plastic flow beneath the footing, thereby providing a measure of the ability of *UDEC* to model this condition.

A strip footing is evaluated to demonstrate the capability of *UDEC* to predict collapse loads and model plastic flow of intact material. The strip footing has a rough base with a width of 6.0 m, and is located on a frictionless, cohesive soil that has the following properties:

density (ρ)	1000 kg/m^3
shear modulus (G)	100 MPa
bulk modulus (K)	200 MPa
cohesion (c)	10 kPa
friction angle (ϕ)	0
dilation angle (ψ)	0

8.2 Analytical Solution

The bearing capacity for a strip footing is from the solution to “Prandtl’s wedge,” as given by Terzaghi and Peck (1967):

$$q = (2 + \pi)c$$

or

$$q = 5.14c \tag{8.1}$$

where c is the cohesion of the material, and q is the bearing capacity stress at failure. The solution is based on the mode of failure, as shown in Figure 8.1.

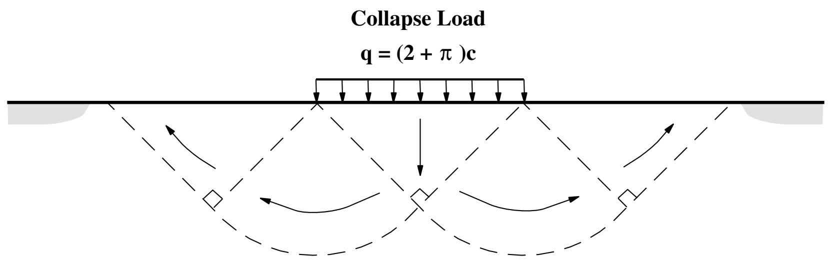

Figure 8.1 Prandtl's wedge problem of a strip footing on a frictionless soil

8.3 *UDEC* Model

A plane-strain analysis is performed for the strip-footing problem. Half-symmetry is used, and boundary conditions are applied, as shown in Figure 8.2.

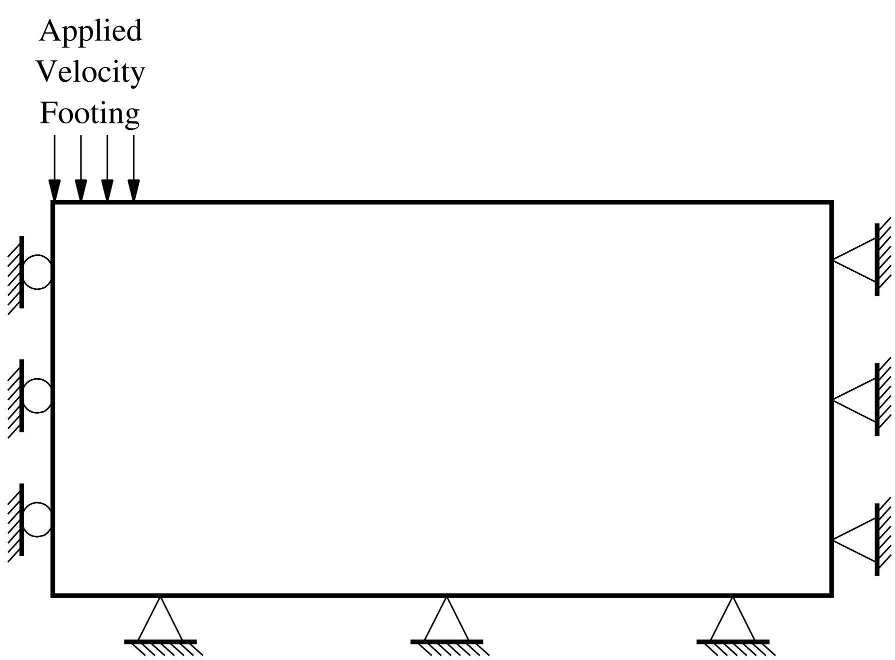

Figure 8.2 UDEC model boundary conditions

Two model grids are created for this problem. The first is a single-block model, and the second is a two-block model created with a diagonal construction joint.

The first model grid, shown in Figure 8.3, is composed of 2048 triangular zones in a diametrically opposed triangular pattern. As discussed in Section 1.2.5 in **Theory and Background**, this zone pattern has been demonstrated to provide reasonable accuracy for calculations involving plastic collapse. This pattern is created with either the command

```
block zone gen edge 0.625
```

or the command

```
block zone gen quad 0.64
```

The second model grid, shown in Figure 8.4, is created by first dividing the model region into two blocks with the command

```
block cut crack (0,0) (20,10) join
```

In this case, only the **block zone generate edge** command can be applied because each block only contains three corners. The command

```
block zone gen edge 0.625
```

creates zoning with boundary gridpoints at the same locations as those in the single-block model. The two-block model contains 3104 zones.

The test is velocity-controlled with a downward velocity of 1.0×10^{-3} m/sec applied to the gridpoints located along the boundary corresponding to the area representing the footing. A zero velocity is applied in the x-direction to represent the rough footing condition. The gridpoint locations of the fixed-velocity boundary condition are indicated on the plots in Figures 8.3 and 8.4.

The footing load is calculated in *FISH* function **`stripload`** by summing the y-direction forces at the footing gridpoints and dividing by the representative footing area. The footing load is monitored as a history for comparison with the bearing capacity calculated from Eq. (8.1).

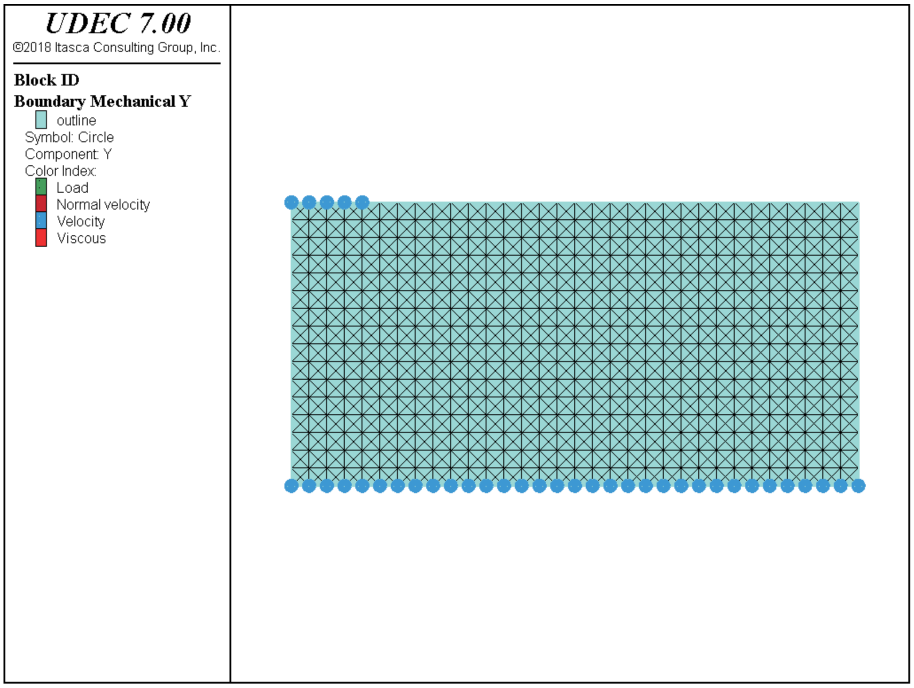

Figure 8.3 UDEC zone geometry for strip footing – single-block model

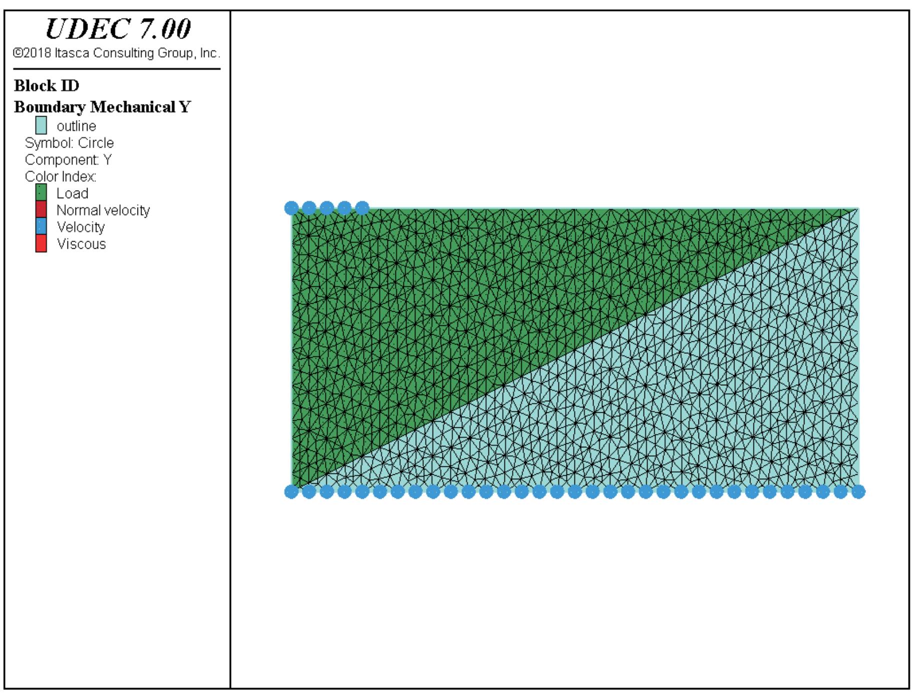

Figure 8.4 UDEC zone geometry for strip footing – two-block model with diagonal construction joint

8.4 Results and Discussion

Figure 8.5 shows the model conditions at the end of the analysis for the single-block model. The behavior shown is very close to that expected from Figure 8.1. Figure 8.6 shows a history of the bearing capacity versus vertical displacement of the footing for the model using **block zone generate quad** zoning. The final value of the bearing capacity for the strip footing is 50.6 kPa, giving an error of 1.66% when compared to the expected value of 51.4 kPa. The results using **block zone generate edge** zoning are essentially identical to those using **block zone generate quad** zoning for the single-block model.

In the two-block model, the triangular zoning has an irregular pattern (as shown in Figure 8.4). This introduces kinematic constraints in the plastic flow calculation and results in an excessively stiff response, as indicated by the bearing capacity history plot in Figure 8.7. The error after 2 cm of vertical settlement of the footing is over 10%, and is increasing.

This problem is discussed in Section 1.2.5 in **Theory and Background**, and can be corrected by applying "nodal mixed discretization" (also described in this section). By adding the command

```
block zone nodal-mixed-discretization on
```

after the **block zone enerate edge** command, nodal mixed discretization is applied to the triangular zoning. The improved result is shown in Figure 8.8. Now the error in the calculated bearing capacity is reduced to 0.8%.

This exercise illustrates that whenever plastic failure and collapse of deformable blocks is to be simulated, the **block zone generate quad** command (or the **block zone generate edge** command with the **block zone nodal-discretization on** command) should be applied in order to obtain an accurate solution.

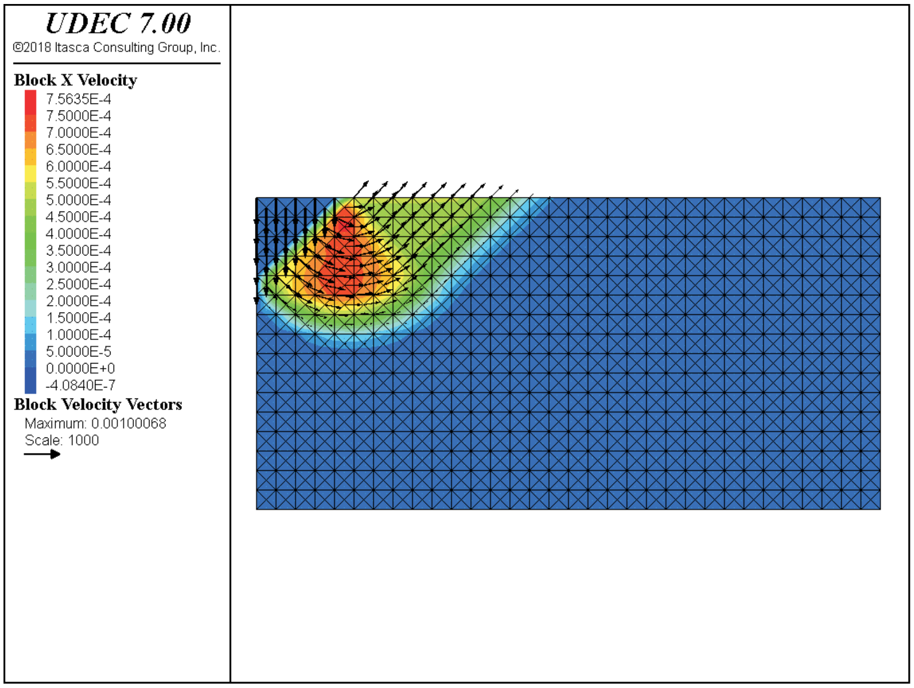

Figure 8.5 Steady state x-velocity contours and velocity vectors at collapse load for strip footing

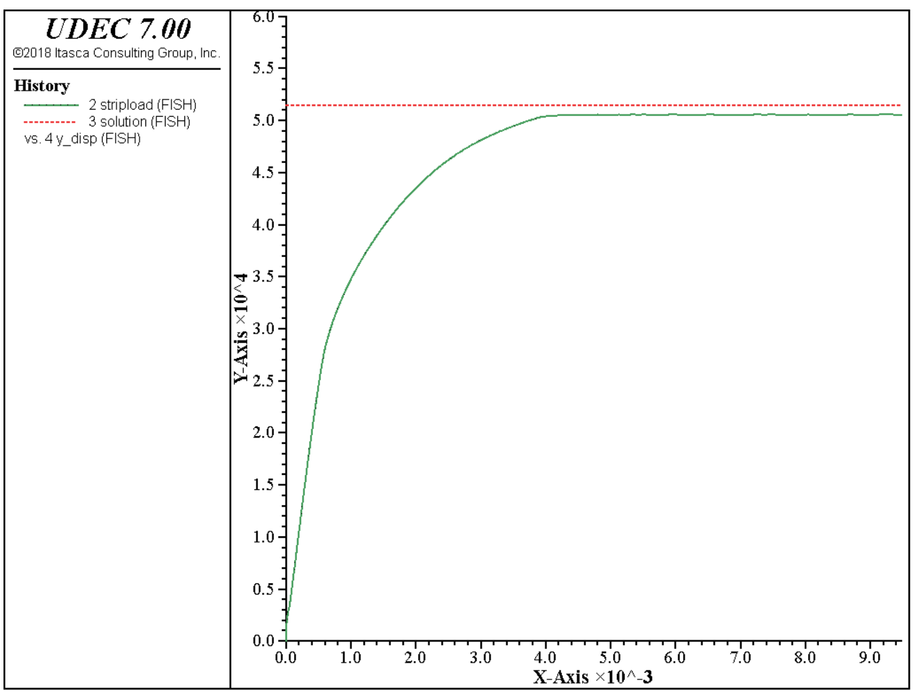

Figure 8.6 History of strip footing load; exact solution also shown – single-block model

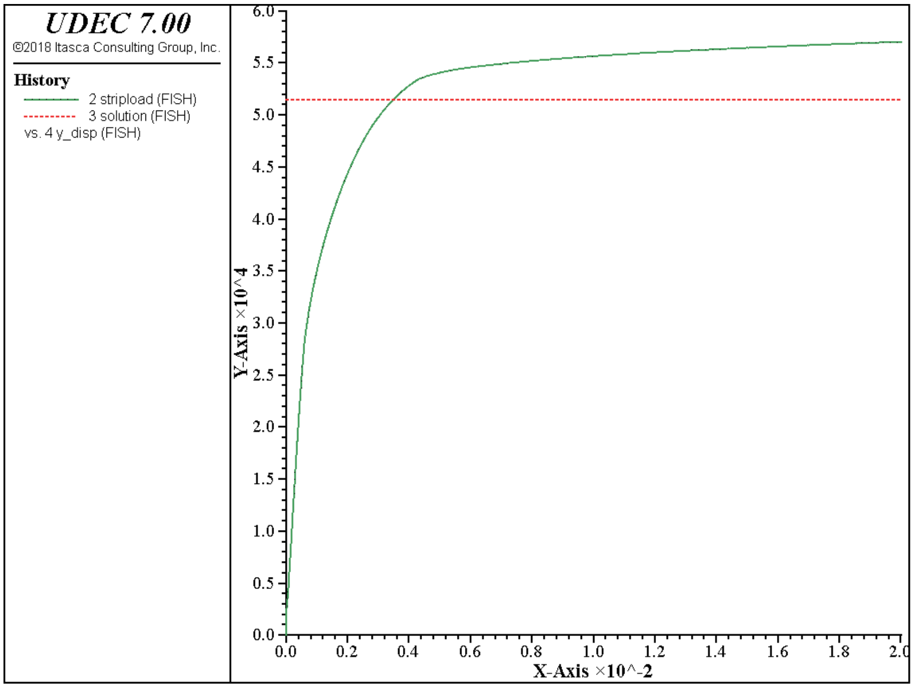

Figure 8.7 History of strip footing load; exact solution also shown – two block model (zoning by **GENERATE edge***)*

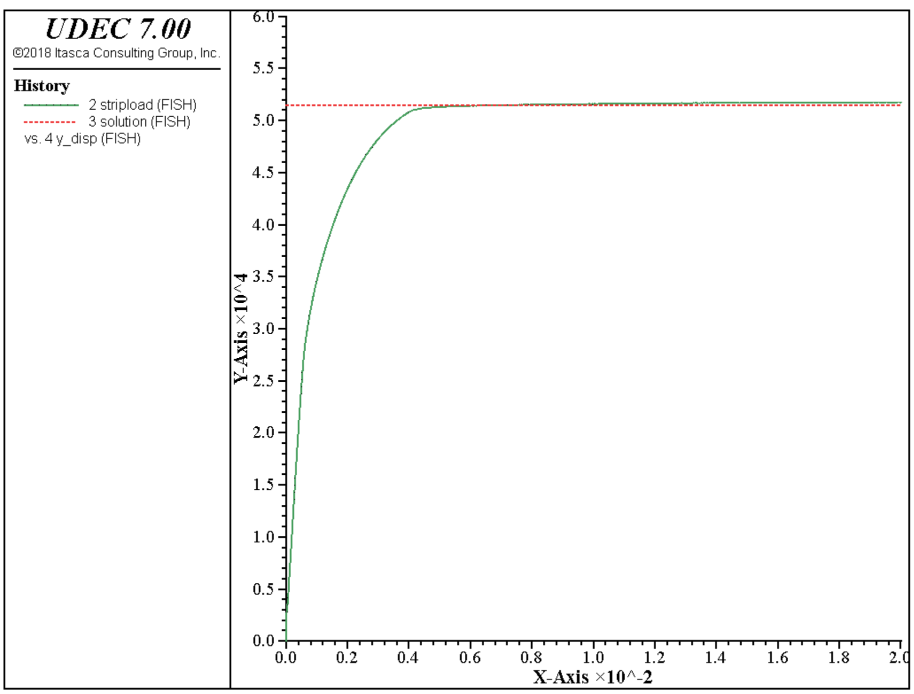

Figure 8.8 History of strip footing load; exact solution also shown – two-block model (zoning by **GENERATE edge** *with* **SET nodal on***)*

8.5 References

Sloan, S. W., and M. F. Randolph. "Numerical Prediction of Collapse Loads Using Finite Element Methods," *Int. J. Num. & Analy. Methods in Geomech.*, ***6***, 47-76 (1982).

Terzaghi, K., and R. B. Peck. ***Soil Mechanics in Engineering Practice***, 2nd Ed. New York: John Wiley and Sons (1967).

8.6 Listing of Data File

Example 8.1 PRAN.DAT

```
model new
;File:pran.dat
;Title:Prandtl’s Wedge Test
; rough footing on cohesive material
block tolerance corner-round-length 0.01
block tolerance minimum-edge-length 0.02
block create polygon 0 0 0 10 20 10 20 0
;
; block zone gen quad zoning for single-block model
block zone gen quad 0.64
;
; block zone gen edge zoning for single-block model and for two-block model
; block zone gen edge 0.625
;
; nodal mixed discretization for two-block model
; set nodat on
;
; material properties
block zone group ’clay’
bl zone cmodel assign mohr-c dens 1E3 bulk 2E8 shear 1E8 coh 1E4 ...
  tens 1E10 range group ’clay’
;
; boundary conditinos
bl grid apply velocity-x 0 range position-x -0.1 0.1 position-y -0.1 10.1
bl grid apply velocity-x 0 range position-x 19.9 20.1 position-y -0.1 10.1
bl grid apply velocity-x 0 range position-x -0.1 20.1 position-y -0.1 0.1
bl grid apply velocity-y 0 range position-x -0.1 20.1 position-y -0.1 0.1
bl grid apply velocity-y -0.001 range position-x -0.1 3 position-y 9.9 10.1
bl grid apply velocity-x 0 range position-x -0.1 3 position-y 9.9 10.1
;
; comparison to analytical solution
fish define p_cons
   p_xp = block.gp.near(3.12,10.0)
   p_xm = block.gp.near(2.50,10.0)
   p_y0 = block.gp.near(0.0,10.0)
   solution=(2.0 + math.pi)*1e4
end
@p_cons
;
fish define stripload
   sum =0.0
   ib = block.head
```

```
    loop while ib # 0
       ig = block.gp(ib)
       loop while ig # 0
          if block.gp.pos.y(ig) > 9.8 then
             if block.gp.pos.x(ig) < 3.0 then
                ibou=block.gp.boundary.corner(ig) ; index of boundary corner
                if(ibou) > 0 then ; exterior boundary
                  forcey = block.boundary.force.y(ibou) ; total y-force
                  sum = sum - forcey
                endif
             endif
          endif
          ig = block.gp.next(ig)
       endloop
       ib = block.next(ib)
    endloop
    x_p  = block.gp.pos.x(p_xp)
    x_m  = block.gp.pos.x(p_xm)
    p_load = 2.0 * sum / (x_p + x_m)
    y_disp = -block.gp.disp.y(p_y0)
    stripload   = p_load
    err    = (p_load-solution)/solution
 end
 @stripload
 fish history @err
 fish history @stripload
 fish history @solution
 fish history @y_disp
 block  smallstrain
 model save 'strip1.sav'
 ;
 block cycle 35000
 model save 'strip2.sav'
```