2 CONTINUOUSLY YIELDING JOINT MODEL

2.1 Background

Numerical modeling of practical problems may take joints through rather complex load paths. Empirical models developed to fit laboratory tests only provide responses to simple loading conditions. More general situations require either interpolation between curves or other arbitrary assumptions. The continuously yielding joint model, proposed by Cundall and Hart (1984), is intended to simulate, in a simple fashion, the internal mechanism of progressive damage of joints under shear. The model also provides continuous hysteretic damping for dynamic simulations by using a "bounding surface" concept similar to that proposed by Dafalias and Herrmann (1982) for soils. An application of the model to study rock-fault instability induced by seismic events is reported by Cundall and Lemos (1990).

2.2 Formulation

The continuously yielding model is considered more "realistic" than the standard Mohr-Coulomb joint model in that the continuously yielding model attempts to account for some nonlinear behavior observed in physical tests (such as joint shearing damage, normal stiffness dependence on normal stress, and decrease in dilation angle with plastic shear displacement). The essential features of the continuously yielding model include the following.

1. The curve of shear stress/shear displacement is always tending toward a "target" shear strength for the joint (i.e., the instantaneous gradient of the curve depends directly on the difference between strength and stress).
2. The target shear strength decreases continuously as a function of accumulated plastic displacement (a measure of damage).
3. Dilation angle is taken as the difference between the apparent friction angle (determined by the current shear stress and normal stress) and the residual friction angle.

As a consequence of these assumptions, the model exhibits, automatically, the commonly observed peak/residual behavior of rock joints. Also, hysteresis is displayed for unloading and reloading cycles of all strain levels, no matter how small.

The model is described as follows. The response to normal loading is expressed incrementally as

$$\Delta\sigma_n = k_n \, \Delta u_n \, , \tag{2.1}$$

where the normal stiffness, k_n, is given by

$$k_n = a_n \, \sigma_n^{e_n} \, , \tag{2.2}$$

representing the observed increase of stiffness with normal stress, where a_n and e_n are model parameters. In general, zero tensile strength is assumed.

For shear loading, the model displays irreversible nonlinear behavior from the onset of shearing. Figure 2.1 shows a typical stress-displacement curve for monotonic loading under constant normal stress. The shear stress increment is calculated as

$$\Delta\tau = F k_s \, \Delta u_s \, , \tag{2.3}$$

where the shear stiffness, k_s, can also be taken as a function of normal stress. For example,

$$k_s = a_s \, \sigma_n^{e_s} . \tag{2.4}$$

The stiffness functions defined by Eqs. (2.2) and (2.4) are the simplest functions consistent with the experimental data. More complex functions (such as hyperbolic laws) may be substituted, if desired.

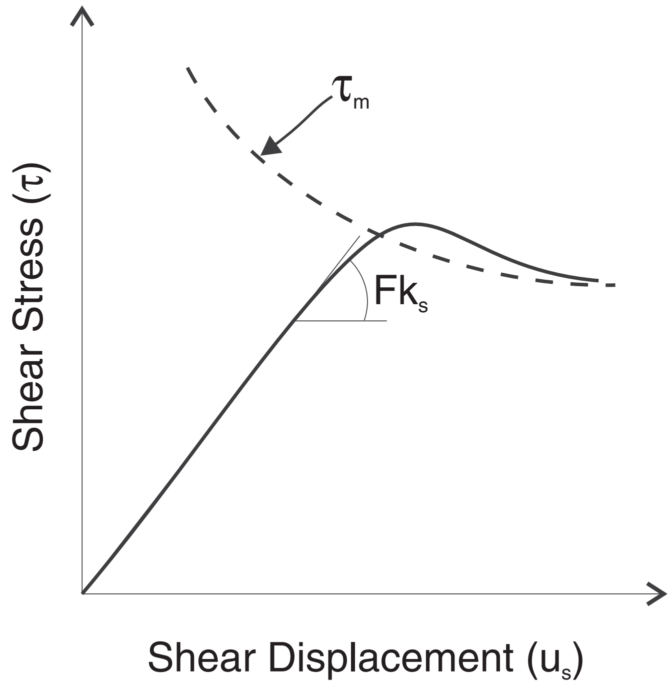


Figure 2.1 Continuously yielding joint model: shear stress-displacement curve and bounding shear strength

The tangent modulus is governed in Eq. (2.3) by the factor F, which depends on the distance from the actual stress curve to the target or bounding strength curve, τ_m, as shown in Figure 2.1.

$$F = \frac{(1 - \tau/\tau_m)}{1 - r} \tag{2.5}$$

The factor r, which is initially zero, is intended to restore the elastic stiffness immediately after a load reversal – that is, r is set to τ/τ_m (and, therefore, F to 1) whenever sgn(Δu_s) is not equal to sgn($\Delta u_s^{(old)}$). In practice, r is limited to 0.75 in order to avoid numerical noise when the shear stress is approximately equal to the bounding strength. The bounding strength is given by

$$\tau_m = \sigma_n \tan \phi_m \operatorname{sgn}(\Delta u_s) \tag{2.6}$$

The parameter ϕ_m can be understood as the friction angle that would apply if the joint were to dilate at the maximum dilation angle. As damage accumulates, this angle is continuously reduced according to the equation

$$\Delta\phi_m = -1/R\ (\phi_m - \phi)\ \Delta u_s^p\ , \tag{2.7}$$

where the plastic displacement increment is defined as

$$\Delta u_s^p = (1 - F)\ |\Delta u_s| \tag{2.8}$$

and ϕ is the basic friction angle of the rock surfaces. R is a material parameter (with dimension of length) that expresses the joint roughness.

ϕ_m can also be thought of as the effective friction angle that would apply if no damage were done (i.e., if no asperities were sheared off). However, before the corresponding strength can be mobilized, some shear displacement is necessary (which reduces ϕ_m).

The parameter R, which has the dimension of length, controls the rate at which ϕ_m decreases with plastic shear displacement. A small value of R causes ϕ_m to decrease rapidly; a large value of R leads to a slower reduction of ϕ_m and, therefore, to a larger peak stress. The peak is reached when the bounding strength equals the shear stress. After this point, the value of F becomes negative and the joint enters the softening regime. In a more complex model, R should depend on σ_n.

The incremental relation for ϕ_m, given by Eq. (2.7), is equivalent to

$$\phi_m = (\phi_m^{(i)} - \phi)\ \exp(-u_s^p/R) + \phi \tag{2.9}$$

where $\phi_m^{(i)}$ is the initial value of ϕ_m, and represents the in-situ state of the joint.

The plastic displacement, $u_s^{(p)}$, always increases. It is evaluated on that part of the applied displacement remaining after the displacement associated with the initial stiffness (given by Eq. (2.4)) is removed. When using the model to match experimental results, the initial value of ϕ_m can be used as a parameter. This corresponds physically to initial pre-shearing or damage of the sample.

The effective dilatancy angle is calculated as

$$i = \tan^{-1}(|\tau|/\sigma_n) - \phi \tag{2.10}$$

(i.e., dilation takes place whenever the stress is above the residual strength level, and is obtained from the actual apparent friction angle). The present formulation may produce unacceptable results when large variations of normal stress accompany reversals in the direction of shearing. For example, consider the case of shearing at a given σ_n, followed by a substantial reduction in normal stress without shear motion, and then by shearing in the opposite direction. If the change in σ_n causes a large drop in the bounding strength, or a stress-dependent shear stiffness is used, then in principle it is possible for the unloading curve to be above the loading curve, leading to energy production. Further research and modifications of the model are required to avoid this problem. In the meantime, the model should be used with caution in situations when large reductions in σ_n occur.

2.3 Summary of Continuously Yielding Model Parameters

The model parameters associated with the continuously yielding model are summarized in Table 2.1. The model is accessed in *UDEC* with the **block contact change model = 3** command or the **block contact cmodel assign continuously-yielding** command.

Table 2.1 Parameters associated with continuously yielding joint model

Parameter	Description
a_n	joint normal stiffness (input)
e_n	joint normal stiffness exponent (input)
a_s	joint shear stiffness (input)
e_s	joint shear stiffness exponent (input)
R	joint roughness parameter (input)
$\phi_m^{(i)}$	joint initial friction angle (input)
ϕ	intrinsic friction angle
ϕ_m	effective friction angle
k_n	normal stiffness defined as a function of σ_n
k_s	shear stiffness defined as a function of σ_n
τ	shear stress on the joint
τ_m	failure or “bounding” shear stress
Δu_s	current shear displacement increment
$\Delta u_s^{(old)}$	previous shear displacement increment
$u_s^{(p)}$	accumulated plastic shear displacement
r	the stress ratio at the last reversal ($r = 0$, initially)
i	effective dilation angle

2.4 Example Applications

The use of the continuously yielding joint model is demonstrated in the following example applications for direct shear testing.

The effect of two different assumptions concerning $\phi_m^{(i)}$ (the joint initial friction (**friction-initial**) angle) are examined. In the first example, the initial friction angle is assumed to be 59.3°; in the second, the joint initial friction angle is taken to be 40.1°. The following problem parameters, listed in Table 2.2, apply to both examples.

Table 2.2 Continuously yielding model properties for direct shear tests

Property Keyword	Description	Value
density	ρ (block mass density)	2600 kg/m^3
bulk	K (bulk modulus of block)	4 GPa
shear	G (shear modulus of block)	3 GPa
st-n	a_n (joint normal stiffness)	100 GPa/m
st-s	a_s (joint shear stiffness)	100 GPa/m
exp-n	e_n (joint normal stiffness exponent)	0.0
exp-s	e_s (joint shear stiffness exponent)	0.0
fric	ϕ (joint intrinsic friction angle)	30°
rough	R (joint roughness parameter)	0.1 mm

Figure 2.2 shows the model for the direct shear tests. A constant normal stress of 10 MPa is applied on the joint first. Then, the top block is moved at a constant horizontal velocity. A *FISH* function, **`av_str`**, is used to calculate the average normal and shear stresses and normal and shear displacements along the joint. The data files for the two tests are given in Examples 2.1 and 2.2.

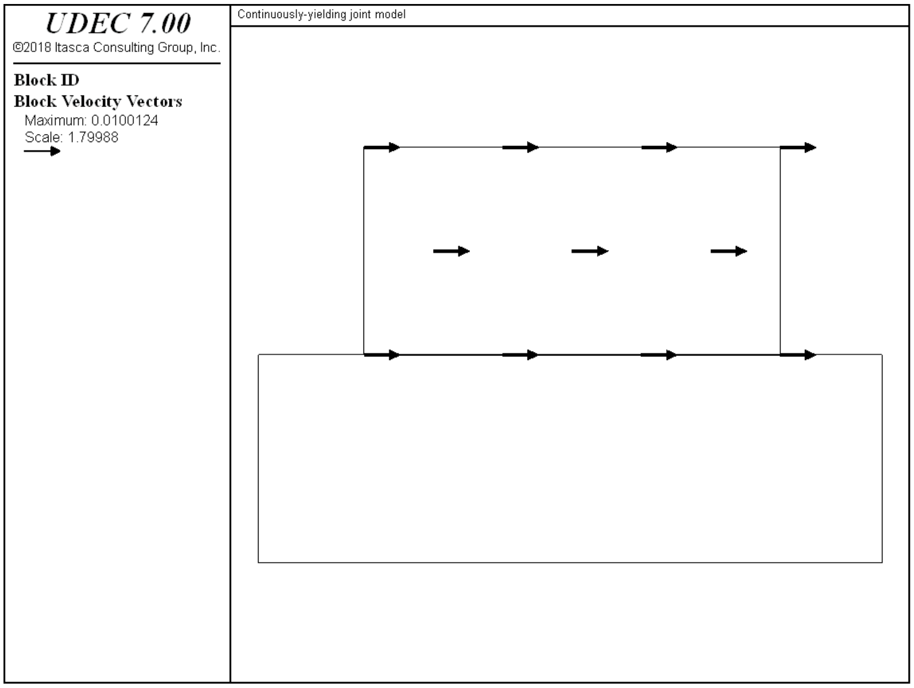
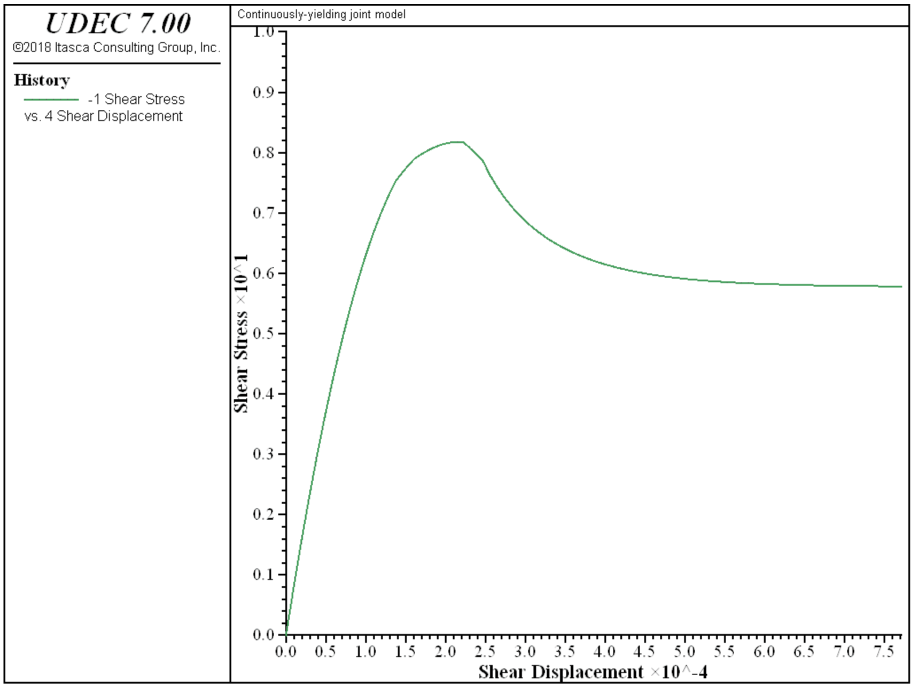
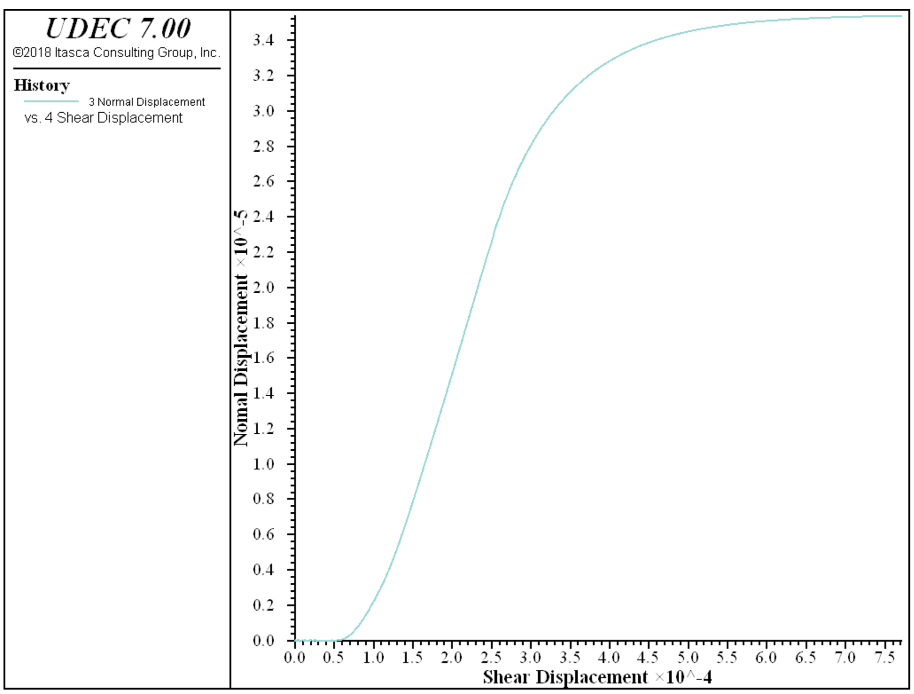


Figure 2.2 UDEC model for direct shear test

Figure 2.3(a) shows a plot of shear stress versus shear displacement for the case in which the joint initial friction was 59.3°. The normal displacement versus shear displacement plot for this assumption is shown in Figure 2.3(b). Similar plots of shear stress and normal displacement versus shear displacement for joint initial friction = 40.1° are given in Figures 2.4(a) and (b).

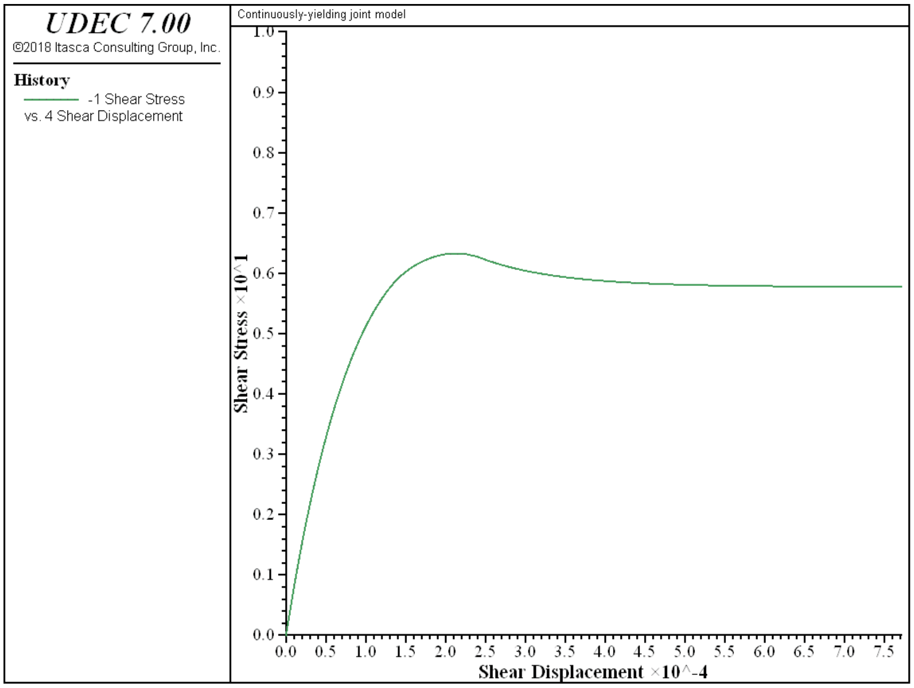


(a) plot of shear stress (MPa) vs shear displacement (m)

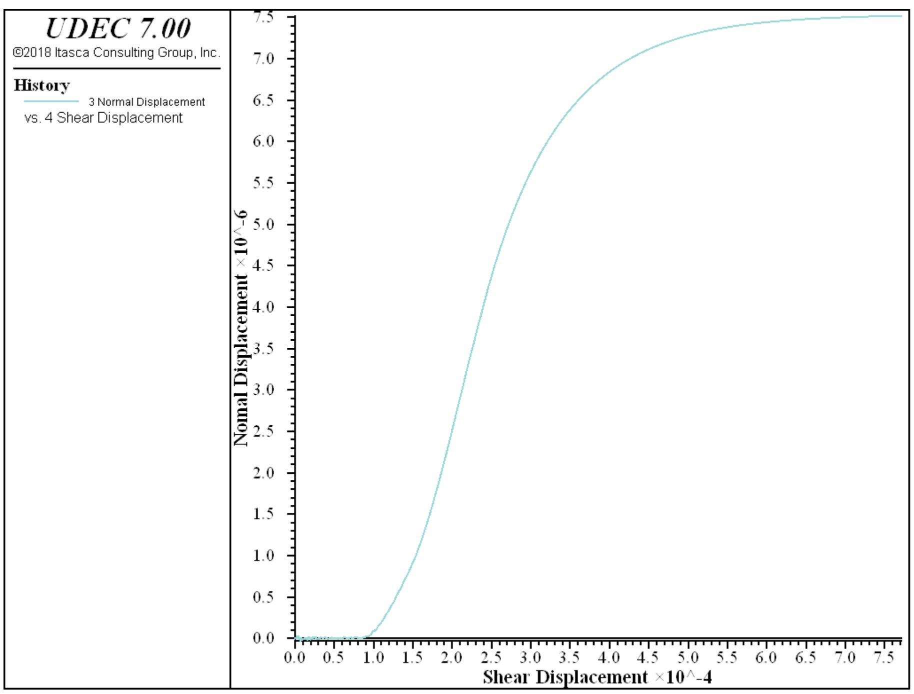


(b) plot of normal displacement (m) vs shear displacement (m)

Figure 2.3 Results of direct shear test using continuously yielding model with initial joint friction at 59.3°



(a) plot of shear stress (MPa) vs shear displacement (m)



(b) plot of normal displacement (m) vs shear displacement (m)

Figure 2.4 Results of direct shear test using continuously yielding model with initial joint friction at 40.1°

Example 2.1 Direct shear test with high peak shear stress

```
model new
;file 'cy_1.dat'
model title 'Continuously-yielding joint model'
;   direct shear test
;   run 1 : high peak stress
;
config udm
block tolerance corner-round-length 0.001
block create polygon -0.05 -0.1 -0.05 0.1 0.25 0.1 0.25 -0.1
block cut crack -1 0 1 0
block cut crack 0 0.1 0 0
block cut crack 0.2 0.1 0.2 0
block delete range pos-x -0.05 0 pos-y 0 0.1
block delete range pos-x 0.2 0.25 pos-y 0 0.1
;
block zone gen quad 0.4 0.11 range pos-x 0 1 pos-y -1 0
block zone gen quad 0.07 0.11 range pos-x 0 1 pos-y 0 1
;
block property material 1 density 2.60e-3
block property material 1 bulk 4000 shear 3000
block domain property material 1 capillary-gamma 3000
;
; C-Y joint model (JOINT model cy)
block contact cmodel assign continuously-yielding
block contact cmodel default continuously-yielding
block contact property stiffness-normal 100000 stiffness-shear 100000 ...
  exponent-normal 0.0 exponent-shear 0.0 friction 30 ...
  friction-initial 59.3 roughness 1.0e-4
;
; apply boundary conditions
block gridpoint apply velocity-x 0 range pos-x -0.06 -0.04 pos-y -1 1
block gridpoint apply velocity-x 0 range pos-x 0.24 0.26 pos-y -1 1
block gridpoint apply velocity-y 0 range pos-x -1 1 pos-y -0.11 -0.09
; apply normal load
block edge apply stress 0 0 -10 range pos-x -1 1 pos-y 0.09 0.11
;
;
block cycle 1000
;
; functions to calculate average joint stresses
; and average joint displacements
;
fish define ini_jdisp
```

```
    njdisp0 = 0.0
    sjdisp0 = 0.0
    ic = block.contact.head
    loop while ic # 0
        njdisp0 = njdisp0 + block.contact.disp.normal(ic)
        sjdisp0 = sjdisp0 + block.contact.disp.shear(ic)
        ic = block.contact.next(ic)
    endloop
  end
  @ini_jdisp
  ;
  fish define av_str
    whilestepping
    sstav = 0.0
    nstav = 0.0
    njdisp = 0.0
    sjdisp = 0.0
    ncon = 0
    jl   = 0.2              ; joint length
    ic = block.contact.head
    loop while ic # 0
        ncon = ncon+1
        sstav = sstav + block.contact.force.shear(ic)
        nstav = nstav + block.contact.force.normal(ic)
        njdisp = njdisp + block.contact.disp.normal(ic)
        sjdisp = sjdisp + block.contact.disp.shear(ic)
        ic = block.contact.next(ic)
    endloop
    if ncon # 0
      sstav = sstav / jl
      nstav = nstav / jl
      njdisp = (njdisp-njdisp0) / ncon
      sjdisp = (sjdisp-sjdisp0) / ncon
    endif
  end
  ;
  block contact reset displacement
  block gridpoint init displacement-x 0
  block gridpoint init displacement-y 0
  hist reset
  ;
  hist  interval 1
  fish history @sstav
  fish history @nstav
  fish history @njdisp
  fish history @sjdisp
```

```
hist name 1 label 'Shear Stress'
hist name 3 label 'Normal Displacement'
hist name 4 label 'Shear Displacement'
;
; apply shear load by imposing x-velocity on top block
block gridpoint apply velocity-x 0.01 range pos-x -.01 .21 pos-y -.01 .11
;
block cycle 6500
;
;plot hold hist -1 vs 4
;plot hold hist 3 vs 4
;
model save 'cy_1.sav'
;
return
```

Example 2.2 Direct shear test with low peak shear stress

```
model new
;file: cy_2.dat
;   continuously-yielding joint model
;   direct shear test
;   run 2 : low peak stress
;
block config udm
block tolerance corner-round-length 0.001
block create polygon (-0.05,-0.1)  (-0.05,0.1)  (0.25,0.1)  (0.25,-0.1)
block cut crack -1 0 1 0
block cut crack 0 0.1 0 0
block cut crack 0.2 0.1 0.2 0
block delete range pos-x -0.05 0 pos-y 0 0.1
block delete range pos-x 0.2 0.25 pos-y 0 0.1
;
block zone gen quad 0.4 0.11 range pos-x 0 1 pos-y -1 0
block zone gen quad 0.07 0.11 range pos-x 0 1 pos-y 0 1
;
block property material 1 density 2.60e-3 bulk 4000 shear 3000
;
; C-Y joint model (JOINT model cy)
block contact cmodel assign continuously-yielding
block contact cmodel default continuously-yielding
block contact property stiffness-normal 100000 stiffness-shear 100000 ...
  exponent-normal 0.0 exponent-shear 0.0 friction 30 ...
  friction-initial 40.1 roughness 1.0e-4
;
```

```
; apply boundary conditions
block gridpoint apply velocity-x 0 range pos-x -0.06 -0.04 pos-y -1 1
block gridpoint apply velocity-x 0 range pos-x 0.24 0.26 pos-y -1 1
block gridpoint apply velocity-y 0 range pos-x -1 1 pos-y -0.11 -0.09
; apply normal load
block edge apply stress 0 0 -10 range pos-x -1 1 pos-y 0.09 0.11
;
;
block cycle 1000
;
; functions to calculate average joint stresses
; and average joint displacements
;
fish define ini_jdisp
  njdisp0 = 0.0
  sjdisp0 = 0.0
  ic = block.contact.head
  loop while ic # 0
      njdisp0 = njdisp0 + block.contact.disp.normal(ic)
      sjdisp0 = sjdisp0 + block.contact.disp.shear(ic)
      ic = block.contact.next(ic)
  endloop
end
@ini_jdisp
;
fish define av_str
  whilestepping
  sstav = 0.0
  nstav = 0.0
  njdisp = 0.0
  sjdisp = 0.0
  ncon = 0
  jl   = 0.2              ; joint length
  ic = block.contact.head
  loop while ic # 0
      ncon = ncon+1
      sstav = sstav + block.contact.force.shear(ic)
      nstav = nstav + block.contact.force.normal(ic)
      njdisp = njdisp + block.contact.disp.normal(ic)
      sjdisp = sjdisp + block.contact.disp.shear(ic)
      ic = block.contact.next(ic)
  endloop
  if ncon # 0
    sstav = sstav / jl
    nstav = nstav / jl
    njdisp = (njdisp-njdisp0) / ncon
```

```
    sjdisp = (sjdisp-sjdisp0) / ncon
  endif
end
;
block contact reset displacement
block gridpoint init displacement-x 0
block gridpoint init displacement-y 0
hist reset
;
hist interval 1
fish history @sstav
fish history @nstav
fish history @njdisp
fish history @sjdisp
hist name 1 label 'Shear Stress'
hist name 3 label 'Normal Displacement'
hist name 4 label 'Shear Displacement'
;
; apply shear load by imposing x-velocity on top block
block gridpoint apply velocity-x 0.01 range pos-x -.01 .21 pos-y -.01 .11
;
block cycle 6500
;
;plot hold hist 1 vs 4 yr
;plot hold hist 3 vs 4
;
model save 'cy_2.sav'
;
return
```

2.5 References

Cundall, P. A., and R. D. Hart. "Analysis of Block Test No. 1 Inelastic Rock Mass Behavior: Phase 2 – A Characterization of Joint Behavior (Final Report)." Itasca Consulting Group Report, Rockwell Hanford Operations, Subcontract SA-957 (1984).

Cundall, P. A., and J. V. Lemos. "Numerical Simulation of Fault Instabilities with a Continuously Yielding Joint Model," in ***Rockbursts and Seismicity in Mines***, pp. 147-152. C. Fairhurst, ed. Rotterdam: A. A. Balkema (1990).

Dafalias, Y. F., and L. R. Herrmann. "Bounding Surface Formulation of Soil Plasticity," in ***Soil Mechanics – Transient and Cyclic Loads***, Vol. 10, pp. 253-282. Chichester: John Wiley & Sons (1982).