1 CREEP MATERIAL MODELS

1.1 Introduction

These models can be used to simulate the behavior of materials that exhibit creep (i.e., time-dependent material behavior). The **block config creep** command must precede the use of these models. Eight creep models have been implemented in *UDEC*:

(1) a classical viscoelastic model (**block zone cmodel assign maxwell**);

(2) a Burgers substance viscoelastic model (**block zone cmodel assign burgers**);

(3) a two-component power law (**block zone cmodel assign power**);

(4) a reference creep formulation (the WIPP model) for nuclear-waste isolation studies (**block zone cmodel assign wipp**);

(5) a Burgers-creep viscoplastic model combining the Burgers model and the Mohr-Coulomb model (**block zone cmodel assign burger-mohr**);

(6) a power-law viscoplastic model combining the two-component power law and the Mohr-Coulomb model (**block zone cmodel assign power-mohr**);

(7) a WIPP-creep viscoplastic model combining the WIPP model and the Drucker-Prager model (**block zone cmodel assign wipp-drucker**); and

(8) a crushed-salt constitutive model (**block zone cmodel assign wipp-salt**).

The models are presented in order of increasing complexity. The first model is the classical formulation known as the Maxwell substance, and the second is the classical formulation for a Burgers substance. The third model can be used for mining applications (e.g., salt or potash mining), and the fourth model is commonly used in thermomechanical analyses associated with studies for the underground isolation of nuclear waste in salt. The fifth model expands on the second model and also includes a Mohr-Coulomb component. The sixth model is a variation of the third model, and includes a Mohr-Coulomb component. The seventh model is a variation of the fourth model, and includes a Drucker-Prager plasticity component. The eighth model is also a variation of the fourth, and includes volumetric and deviatoric compaction behavior. Descriptions of these models and their implementation are provided in this section.*

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* The data files in this section are all created in a text editor. The files are stored in the directory "ITASCA\UDEC700\Datafiles\Creep" with the extension ".DAT." A project file is also provided for each example. In order to run an example and compare the results to plots in this section, first copy the files to a folder where you have write access, open a project file in the *GIIC* by clicking on the FILE/OPEN PROJECT menu item and selecting the project file name (with extension ".PRJ"). Click on the *Project Options* icon at the top of the *Project Tree Record*, select *Rebuild unsaved states*, and the example data file will be run, and plots created.

1.2 Description of Creep Constitutive Models

1.2.1 Classical Viscoelasticity (Maxwell Substance)

The classical notion of Newtonian viscosity is that the rate of strain is proportional to stress. Stress-strain relations can be developed for viscous flow in a way similar to the way relationships are developed for elastic deformation. The derivation of the equations in three dimensions can be found, for example, in Jaeger (1969).

Viscoelastic materials exhibit both viscous and elastic behaviors. One such material is the Maxwell material, which can be represented in one dimension by a spring (with elastic constant k) in series with dashpot (of viscous constant η). The incremental force/displacement law for this material can be written as

$$\dot{u} = \frac{\dot{F}}{k} + \frac{F}{\eta} \tag{1.1}$$

where $\dot{u}$ is the velocity, and F is the force. Denoting the new value of force by F', and the old value by F°, over a timestep of Δt, we can rewrite Eq. (1.1) as

$$\frac{\Delta u}{\Delta t} = \frac{F' - F^{\circ}}{k\Delta t} + \frac{F' + F^{\circ}}{2\eta} \tag{1.2}$$

This is a central difference equation, since the velocity is calculated at the midpoint between the instances when F' and F° are defined. Solving for F',

$$F' = (F^{\circ} C_1 + k\Delta u)\, C_2 \tag{1.3}$$

where

$$C_1 = 1 - \frac{k\Delta t}{2\eta}$$

$$C_2 = \frac{1}{1 + \frac{k\Delta t}{2\eta}}$$

An equation identical to Eq. (1.3) can be written for the relation between deviatoric stresses and strain increments:

$$\sigma_{ij}^{d} = (\sigma_{ij}^{d^{\circ}} C_1 + 2G\ \Delta\epsilon_{ij}^{d})\, C_2 \tag{1.4}$$

where

$$\Delta\epsilon_{ij}^{d} = \Delta\epsilon_{ij} - \frac{1}{3}\,\Delta\epsilon_{ij}\,\delta_{ij}$$

$$\sigma_{ij}^{d^{\circ}} = \sigma_{ij}^{\circ} - \frac{1}{3}\,\sigma_{ij}^{\circ}\,\delta_{ij}$$

$$C_1 = 1 - \frac{G\Delta t}{2\eta}$$

$$C_2 = \frac{1}{1 + \frac{G\Delta t}{2\eta}}$$

Here, $\Delta\epsilon_{ij}$ are the components of the "input" strain-increment tensor, σ_{ij}° are the components of the previous stress tensor, and G is the shear modulus. For the volumetric component of stress and strain, we assume that there are no viscous effects (elastic relations apply):

$$\sigma^{\text{iso}} = \frac{1}{3}\,\sigma_{kk}^{\circ} + K\,\Delta\epsilon_{kk} \tag{1.5}$$

where K is the bulk modulus. The final stress tensor is given by the sum of the deviatoric and isotropic parts:

$$\sigma_{ij} = \sigma_{ij}^{d} + \sigma^{\text{iso}}\delta_{ij} \tag{1.6}$$

The material properties required for this model are shear and bulk moduli (for the elastic behavior) and the viscosity. Under an applied shear stress, the material flows continuously, but it behaves elastically under an applied isotropic stress.

1.2.2 Burgers Model

The Burgers model is composed of a Kelvin model and a Maxwell model connected in series (see Figure 1.1 for symbol definitions). The equations for the Kelvin sub-model are

$$\dot{u}_k = \frac{F_d}{\eta_1} \tag{1.7}$$

$$F_d = F - k_1 u_k \tag{1.8}$$

Combining Eqs. (1.7) and (1.8) in finite-difference form,

$$u'_k = u^\circ_k + \left(\bar{F} - k_1 \bar{u}_k\right) \frac{\Delta t}{\eta_1} \tag{1.9}$$

where $\bar{F}$ and $\bar{u}_k$ correspond to mean values of F and u_k over the timestep, and the superscripts $'$ and $^\circ$ denote new and old values, respectively. Hence,

$$u'_k = u^\circ_k + \left\{F' + F^\circ - k_1(u'_k + u^\circ_k)\right\} \frac{\Delta t}{2\eta_1} \tag{1.10}$$

The equation for the Maxwell sub-model is

$$\dot{u}_m = \frac{\dot{F}}{k_2} + \frac{\bar{F}}{\eta_2} \tag{1.11}$$

which becomes

$$u'_m = u^\circ_m + \frac{F' - F^\circ}{k_2} + \left\{\frac{F' + F^\circ}{2\eta_2}\right\} \Delta t \tag{1.12}$$

when expressed in finite-difference form. Finally, the Kelvin and Maxwell displacement increments combine to give the applied displacement increment,

$$u' - u^\circ = u'_m - u^\circ_m + u'_k - u^\circ_k \tag{1.13}$$

The unknowns in Eqs. (1.10), (1.12) and (1.13) are u'_k, u'_m and F', and the known values are u°_k and F°. The response of the Burgers model is dependent on past history; the state variable that records history information is u_k, which has an evolution equation derived from Eq. (1.10):

$$u'_k = \frac{1}{A}\left\{Bu_k^\circ + \left(F' + F^\circ\right)\frac{\Delta t}{2\eta_1}\right\} \tag{1.14}$$

where

$$A = 1 + \frac{k_1 \Delta t}{2\eta_1} \tag{1.15}$$

$$B = 1 - \frac{k_1 \Delta t}{2\eta_1} \tag{1.16}$$

By combining Eqs. (1.12) and (1.13), and substituting u'_k from Eq. (1.14), we obtain

$$F' = \frac{1}{X}\left\{u' - u^\circ + YF^\circ - \left(\frac{B}{A} - 1\right)u_k^\circ\right\} \tag{1.17}$$

where

$$X = \frac{1}{k_2} + \frac{\Delta t}{2\eta_2} + \frac{\Delta t}{2A\eta_1} \tag{1.18}$$

$$Y = \frac{1}{k_2} - \frac{\Delta t}{2\eta_2} - \frac{\Delta t}{2A\eta_1} \tag{1.19}$$

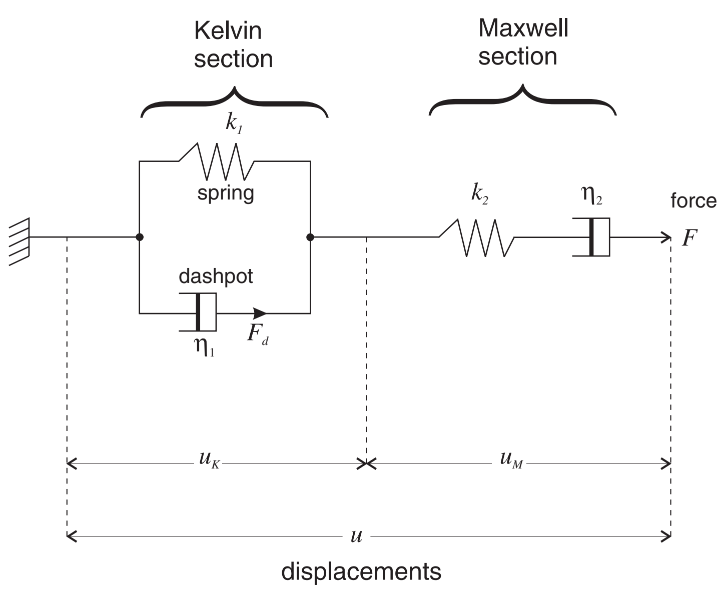

Figure 1.1 Schematic of the Burgers model, with definition of variables

1.2.3 The Two-Component Power Law

The Norton power law (Norton 1929) is commonly used to model the creep behavior of salt. The standard form of this law is

$$\dot{\epsilon}_{cr} = A\,\bar{\sigma}^n \tag{1.20}$$

where $\dot{\epsilon}_{cr}$ is the creep rate, A and n are material properties, $\bar{\sigma} = \left(\frac{3}{2}\right)^{1/2}\left(\sigma^d_{ij}\,\sigma^d_{ij}\right)^{1/2}$, with σ^d_{ij} being the deviatoric part of σ_{ij}.

The deviatoric stress increments are given by

$$\Delta\sigma^d_{ij} = 2G(\dot{\epsilon}^d_{ij} - \dot{\epsilon}^c_{ij})\Delta t \tag{1.21}$$

where G is the shear modulus, and $\dot{\epsilon}^d_{ij}$ is the deviatoric part of the strain-rate tensor.

The creep strain-rate tensor is calculated as

$$\dot{\epsilon}^c_{ij} = \left(\frac{3}{2}\right)\dot{\epsilon}_{cr}\left(\frac{\sigma^d_{ij}}{\bar{\sigma}}\right) \tag{1.22}$$

with $\dot{\epsilon}_{cr}$ and $\bar{\sigma}$ defined as above.

The volumetric behavior is assumed elastic. The isotropic stress increment is given by

$$\Delta\sigma_{kk} = 3K\,\Delta\epsilon_v \tag{1.23}$$

where K is the bulk modulus, and $\Delta\epsilon_v = \Delta\epsilon_{11} + \Delta\epsilon_{22} + \Delta\epsilon_{33}$.

Usually, the amount of data available does not justify adding any more parameters to the creep law. There are cases, however, in which it is justifiable to use a law based on multiple creep mechanisms. *UDEC*, therefore, includes an option to use a two-component law of the form

$$\dot{\epsilon}_{cr} = \dot{\epsilon}_1 + \dot{\epsilon}_2 \tag{1.24}$$

where

$$\dot{\epsilon}_1 = \begin{cases} A_1\,\bar{\sigma}^{n_1} & \bar{\sigma} \geq \sigma_1^{ref} \\ 0 & \bar{\sigma} < \sigma_1^{ref} \end{cases}$$

$$\dot{\epsilon}_2 = \begin{cases} A_2\,\bar{\sigma}^{n_2} & \bar{\sigma} \le \sigma_2^{ref} \\ \\ 0 & \bar{\sigma} > \sigma_2^{ref} \end{cases}$$

With these two terms, several options are possible:

1. The Default Option

$$\sigma_1^{ref} = \sigma_2^{ref} = 0$$

$\bar{\sigma}$ is always positive, so this is the one-component law with

$$\dot{\epsilon}_{cr} = A_1\,\bar{\sigma}^{n_1} \qquad \bar{\sigma} \ge \sigma_1^{ref}$$

2. Both Components Active

$$\sigma_1^{ref} = 0$$

$$\sigma_2^{ref} = \text{“large”}$$

$$\dot{\epsilon}_{cr} = A_1\,\bar{\sigma}^{n_1} + A_2\,\bar{\sigma}^{n_2} \qquad \sigma_1^{ref} < \bar{\sigma} < \sigma_2^{ref}$$

3. Different Law for Different Stress Regimes

   (a) $\sigma_1^{ref} = \sigma_2^{ref} = \sigma^{ref} > 0$

$$\dot{\epsilon}_{cr} = \begin{cases} A_2\,\bar{\sigma}^{n_2} & \bar{\sigma} < \sigma^{ref} \\ A_1\,\bar{\sigma}^{n_1} & \bar{\sigma} > \sigma^{ref} \end{cases}$$

(b) $\sigma_1^{ref} < \sigma_2^{ref}$

$$\dot{\epsilon}_{cr} = \begin{cases} A_2\,\bar{\sigma}^{n_2} & \bar{\sigma} < \sigma_1^{ref} \\ A_1\,\bar{\sigma}^{n_1} + A_2\,\sigma^{n_2} & \sigma_1^{ref} < \bar{\sigma} < \sigma_2^{ref} \\ A_1\,\bar{\sigma}^{n_1} & \bar{\sigma} > \sigma_2^{ref} \end{cases}$$

(c) $\sigma_1^{ref} > \sigma_2^{ref}$

NOTE: Do *not* use option (c). It implies that creep occurs for $\bar{\sigma} < \sigma_2^{ref}$ and for $\bar{\sigma} > \bar{\sigma}_1^{ref}$, but not for $\sigma_2^{ref} < \bar{\sigma} < \sigma_1^{ref}$.

The two-component power law is implemented in *UDEC* by the following procedure.

Let $\sigma_{ij}^{(t)}$ be the stress tensor at time t, and let $\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{e} + \dot{\epsilon}_{ij}^{c}$ be the strain-rate tensor, which consists of an elastic component ($\dot{\epsilon}_{ij}^{e}$) and a creep component ($\dot{\epsilon}_{ij}^{c}$).

The stress $\sigma_{ij}^{(t+\Delta t)}$ at time $t + \Delta t$, is calculated in the following way.

Volumetric Component –

$$\sigma_{kk}^{(t+\Delta t)} = \sigma_{kk}^{(t)} + 3K\dot{\epsilon}_{kk}\,\Delta t \tag{1.25}$$

Deviatoric Component –

$$\sigma_{ij}^{d(t+\Delta t)} = \sigma_{ij}^{d(t)} + 2G(\dot{\epsilon}_{ij} - \dot{\epsilon}_{ij}^{c})\,\Delta t \tag{1.26}$$

where $\dot{\epsilon}_{ij}^{c}$ is given by Eq. (1.22), and K and G are the elastic bulk and shear moduli.

In *UDEC*, velocities and strain rates are evaluated at mid-step. Therefore, the deviatoric stresses in Eq. (1.22) can be calculated as the average:

$$\sigma_{ij}^{d(t+\frac{\Delta t}{2})} = \frac{1}{2}\left[\sigma_{ij}^{d(t)} + \sigma_{ij}^{d(t+\Delta t)}\right] \tag{1.27}$$

An iteration is performed between Eqs. (1.22) and (1.27) to obtain a better approximation.

1.2.4 A Reference Creep Law for Nuclear-Waste Isolation Studies

An empirical creep law, known as the WIPP-reference creep law, has been developed to describe the time- and temperature-dependent creep of natural rock salt, specifically for nuclear waste isolation studies. The model is described by Herrmann et al. (1980a and b); a different expression of the same creep law is also given by Senseny (1985).

The WIPP-reference creep law, as implemented in *UDEC*, partitions the deviatoric strain-rate tensor, $\dot{\epsilon}^d_{ij}$, into elastic and viscous parts ($\dot{\epsilon}^{de}_{ij}$ and $\dot{\epsilon}^{dv}_{ij}$, respectively):

$$\dot{\epsilon}^d_{ij} = \dot{\epsilon}^{de}_{ij} + \dot{\epsilon}^{dv}_{ij} \tag{1.28}$$

where the deviatoric strain-rate is obtained:

$$\dot{\epsilon}^d_{ij} = \dot{\epsilon}_{ij} - \frac{\dot{\epsilon}_{kk}\delta_{ij}}{3} \tag{1.29}$$

The elastic part is related to the deviatoric stress-rate:

$$\dot{\epsilon}^{de}_{ij} = \frac{\dot{\sigma}^d_{ij}}{2G} \tag{1.30}$$

where G is the elastic shear modulus, and

$$\dot{\sigma}^d_{ij} = \dot{\sigma}_{ij} - \frac{\dot{\sigma}_{kk}\delta_{ij}}{3} \tag{1.31}$$

The viscous part of the deviatoric strain-rate is coaxial with the deviatoric stress tensor (normalized by its magnitude, $\bar{\sigma}$, defined in Eq. (1.36)), and is given by

$$\dot{\epsilon}^{dv}_{ij} = \frac{3}{2}\left\{\frac{\sigma^d_{ij}}{\bar{\sigma}}\right\}\dot{\epsilon} \tag{1.32}$$

where the scalar strain-rate, $\dot{\epsilon}$, is composed of two parts, $\dot{\epsilon}_p$ and $\dot{\epsilon}_s$, corresponding to primary and secondary creep, respectively,

$$\dot{\epsilon} = \dot{\epsilon}_p + \dot{\epsilon}_s \tag{1.33}$$

The formulation for the primary creep rate depends on the magnitude of the secondary creep rate:

$$\dot{\epsilon}_p = \begin{cases} (A - B\epsilon_p)\dot{\epsilon}_s\,, & \text{if } \dot{\epsilon}_s \geq \dot{\epsilon}^*_{ss} \\ \{A - B(\dot{\epsilon}^*_{ss}/\dot{\epsilon}_s)\epsilon_p\}\dot{\epsilon}_s\,, & \text{if } \dot{\epsilon}_s < \dot{\epsilon}^*_{ss} \end{cases} \tag{1.34}$$

The secondary creep rate is

$$\dot{\epsilon}_s = D\,\bar{\sigma}^n e^{(-Q/RT)} \tag{1.35}$$

where D, n, A, B and $\dot{\epsilon}^*_{ss}$ are material constants, R is the universal gas constant, Q is the activation energy, T is the temperature in degrees Kelvin, and $\bar{\sigma}$ is the stress magnitude

$$\bar{\sigma} = \sqrt{\frac{3\,\sigma^d_{ij}\sigma^d_{ij}}{2}} \tag{1.36}$$

The volumetric response of the model is purely elastic, and is given by

$$\dot{\epsilon}_{kk} = \frac{\dot{\sigma}_{kk}}{3K} \tag{1.37}$$

where K is the bulk modulus.

An iterative approach is used to apply the above equations, because the constitutive models in *UDEC* take the components of strain rate as independent variables. A model must supply the new stress tensor, on the assumption of constant strain increments. On the first iteration, the stress components, σ^d_{ij}, are taken to be the current ones; creep strain-rates are computed according to Eq. (1.32). New deviatoric stress components, $\sigma^{d'}_{ij}$, are then computed on the basis of Eqs. (1.28), (1.30) and (1.32) as

$$\sigma^{d'}_{ij} = \sigma^{d^\circ}_{ij} + 2G\,\Delta t\,(\dot{\epsilon}^d_{ij} - \dot{\epsilon}^{dv}_{ij}) \tag{1.38}$$

where $\sigma^{d^\circ}_{ij}$ are the stress components that exist on entry to the constitutive model, and Δt is the creep timestep.

On the next and subsequent iterations, the averages of the new and old stress components are used in the creep equations – i.e.,

$$\sigma_{ij}^{d} = (\sigma_{ij}^{d^\circ} + \sigma_{ij}^{d'})/2 \tag{1.39}$$

Further, the mean primary creep-strain, ϵ_p, is determined during every iteration:

$$\epsilon_p = \epsilon_p^\circ + \dot{\epsilon}_p \Delta t/2 \tag{1.40}$$

and used in Eq. (1.34). The quantity ϵ_p° is the primary creep-strain on entry to the constitutive model; it is updated on exit, as

$$\epsilon_p^\circ := \epsilon_p^\circ + \dot{\epsilon}_p \Delta t \tag{1.41}$$

The WIPP-model notation is summarized and typical values are listed in Table 1.1.

Table 1.1 Notation for the WIPP formulation

WIPP notation	Units	Typical Value
A	—	4.56
B	—	127
D	$Pa^{-n}\ s^{-1}$	5.79×10^{-36}
n	—	4.9
Q	cal/mol	12,000
R	cal/mol K	1.987
$\dot{\epsilon}_{ss}^*$	s^{-1}	5.39×10^{-8}

1.2.5 The Burgers-Creep Viscoplastic Model

A Burgers-creep viscoplastic model in *UDEC* is characterized by a visco-elasto-plastic deviatoric behavior and an elasto-plastic volumetric behavior. The viscoelastic and viscoplastic strain-rate components are assumed to act in series. The viscoelastic constitutive law corresponds to a Burgers model (Kelvin cell in series with a Maxwell component), and the plastic constitutive law corresponds to a Mohr-Coulomb model.

As a notation convention in this section, we use the symbols S_{ij} and e_{ij} to denote deviatoric stress and strain components – i.e.,

$$S_{ij} = \sigma_{ij} - \sigma_0 \delta_{ij} \tag{1.42}$$

$$e_{ij} = \epsilon_{ij} - \frac{e_{vol}}{3} \delta_{ij} \tag{1.43}$$

where

$$\sigma_0 = \frac{\sigma_{kk}}{3} \tag{1.44}$$

and

$$e_{vol} = \epsilon_{kk} \tag{1.45}$$

Also, Kelvin, Maxwell and plastic contributions to stresses and strains are labeled using the superscripts $.^K$, $.^M$ and $.^p$, respectively. With those conventions, the model deviatoric behavior may be described by these relations:

Strain rate partitioning

$$\dot{e}_{ij} = \dot{e}_{ij}^K + \dot{e}_{ij}^M + \dot{e}_{ij}^p \tag{1.46}$$

Kelvin

$$S_{ij} = 2\eta^K \dot{e}_{ij}^K + 2G^K e_{ij}^K \tag{1.47}$$

Maxwell

$$\dot{e}_{ij}^{M} = \frac{\dot{S}_{ij}}{2G^{M}} + \frac{S_{ij}}{2\eta^{M}} \tag{1.48}$$

Mohr-Coulomb

$$\begin{aligned} \dot{e}_{ij}^{p} &= \lambda^{*}\frac{\partial g}{\partial \sigma_{ij}} - \frac{1}{3}\dot{e}_{vol}^{p}\delta_{ij} \\ \dot{e}_{vol}^{p} &= \lambda^{*}\left[\frac{\partial g}{\partial \sigma_{11}} + \frac{\partial g}{\partial \sigma_{22}} + \frac{\partial g}{\partial \sigma_{33}}\right] \end{aligned} \tag{1.49}$$

In turn, the volumetric behavior is given by

$$\dot{\sigma}_0 = K(\dot{e}_{vol} - \dot{e}_{vol}^{p}) \tag{1.50}$$

In those formulas, the properties K and G are the bulk and shear moduli and η is the dynamic viscosity (kinematic viscosity times mass density). The Mohr-Coulomb yield envelope is a composite of shear and tensile criteria. The yield criterion is $f = 0$, and in the principal axes formulation we have:

Shear yielding

$$f = \sigma_1 - \sigma_3 N_{\phi} + 2C\sqrt{N_{\phi}} \tag{1.51}$$

Tension yielding

$$f = \sigma^{t} - \sigma_3 \tag{1.52}$$

where C is the material cohesion, ϕ the friction, $N_{\phi} = (1 + \sin\phi)/(1 - \sin\phi)$, σ^{t} is the tensile strength, and σ_1 and σ_3 are the minimum and maximum principal stresses (compression negative). The potential function g has this form:

Shear failure

$$g = \sigma_1 - \sigma_3 N_{\psi} \tag{1.53}$$

Tension failure

$$g = -\sigma_3 \tag{1.54}$$

where ψ is the material dilation and $N_\psi = (1+\sin\psi)/(1-\sin\psi)$. Finally, λ^* is a parameter that is nonzero during plastic flow only, which is determined by application of the plastic yield condition $f = 0$.

The model implementation closely follows the procedures described in the *UDEC* manual for the Burgers-creep and Mohr-Coulomb models. The principle is to write Eqs. (1.46) to (1.50) in the form of finite increments:

$$\Delta e_{ij} = \Delta e_{ij}^K + \Delta e_{ij}^M + \Delta e_{ij}^p \tag{1.55}$$

$$\overline{S_{ij}}\Delta t = 2\eta^K \Delta e_{ij}^K + 2G^K \overline{e_{ij}}^K \Delta t \tag{1.56}$$

$$\Delta e_{ij}^M = \frac{\Delta S_{ij}}{2G^M} + \frac{\overline{S_{ij}}}{2\eta^M}\Delta t \tag{1.57}$$

$$\Delta\sigma_0 = K(\Delta e_{vol} - \Delta e_{vol}^p) \tag{1.58}$$

where the overbar indicates mean value over the timestep Δt:

$$\overline{S_{ij}} = \frac{S_{ij}^N + S_{ij}^O}{2} \tag{1.59}$$

$$\overline{e_{ij}} = \frac{e_{ij}^N + e_{ij}^O}{2} \tag{1.60}$$

and the superscripts $.^N$ and $.^O$ denote new and old values.

After substitution of Eqs. (1.59) and (1.60) in Eq. (1.56), and solving for $e_{ij}^{K,N}$, the Kelvin strain contribution may be expressed in the form

$$e_{ij}^{K,N} = \frac{1}{A}\left[Be_{ij}^{K,O} + \frac{\Delta t}{4\eta^K}\left(S_{ij}^N + S_{ij}^O\right)\right] \tag{1.61}$$

where

$$A = 1 + \frac{G^K \Delta t}{2\eta^K}$$
$$B = 1 - \frac{G^K \Delta t}{2\eta^K} \tag{1.62}$$

After substitution of Eqs. (1.57) and (1.61) in Eq. (1.55), and solving for the new deviatoric stress component, we find (using the mean value definitions Eqs. (1.59) and (1.60))

$$S_{ij}^N = \frac{1}{a}\left[\Delta e_{ij} - \Delta e_{ij}^p + bS_{ij}^O - (\frac{B}{A} - 1)e_{ij}^{K,O}\right] \tag{1.63}$$

where

$$a = \frac{1}{2G^M} + \frac{\Delta t}{4}\left(\frac{1}{\eta^M} + \frac{1}{A\eta^K}\right)$$
$$b = \frac{1}{2G^M} - \frac{\Delta t}{4}\left(\frac{1}{\eta^M} + \frac{1}{A\eta^K}\right) \tag{1.64}$$

and Eq. (1.61) is used as an evolution law to evaluate $e_{ij}^{K,O}$ in Eq. (1.63). For completeness, Eq. (1.58) is written in the form

$$\sigma_0^N = \sigma_0^O + K(\Delta e_{vol} - \Delta e_{vol}^p) \tag{1.65}$$

In the model implementation in *UDEC*, new trial stress components $\widehat{S_{ij}^N}$ and $\widehat{\sigma_0^N}$ are computed from Eq. (1.63) and Eq. (1.65), assuming viscoelastic increments. Trial principal stress components are calculated and sorted, and the yield function is computed. As long as $f \geq 0$, the trial stresses are taken for new stresses. If $f < 0$, plastic flow is taking place and the trial stresses must be corrected by a component due to incremental plastic strain before their values are assigned to the new stresses and the evolution law is updated. Expressing Eqs. (1.63) and (1.65) in principal axes, we may then write (by definition of trial stresses)

$$S_i^N = \widehat{S_i^N} - \frac{1}{a}\Delta e_i^p$$
$$\sigma_0^N = \widehat{\sigma_0^N} - K\Delta e_{vol}^p \tag{1.66}$$

or, using the definition for deviatoric components:

$$
\begin{aligned}
\sigma_1^N &= \widehat{\sigma_1^N} - \left[\alpha_1 \Delta\epsilon_1^p + \alpha_2(\Delta\epsilon_2^p + \Delta\epsilon_3^p)\right] \\
\sigma_2^N &= \widehat{\sigma_2^N} - \left[\alpha_1 \Delta\epsilon_2^p + \alpha_2(\Delta\epsilon_1^p + \Delta\epsilon_3^p)\right] \\
\sigma_3^N &= \widehat{\sigma_3^N} - \left[\alpha_1 \Delta\epsilon_3^p + \alpha_2(\Delta\epsilon_1^p + \Delta\epsilon_2^p)\right]
\end{aligned}
\tag{1.67}
$$

where

$$
\begin{aligned}
\alpha_1 &= K + \frac{2}{3a} \\
\alpha_2 &= K - \frac{1}{3a}
\end{aligned}
\tag{1.68}
$$

Except for the definitions of α_1 and α_2, these formulas are similar to those obtained in the Mohr-Coulomb model derivation (see Eq. (1.73) in **Constitutive Models**). The plasticity formulation may proceed along similar lines. In doing so, we obtain, for shear yielding:

$$
\begin{aligned}
\sigma_1^N &= \widehat{\sigma_1^N} - \lambda(\alpha_1 - \alpha_2 N_\psi) \\
\sigma_2^N &= \widehat{\sigma_2^N} - \lambda\alpha_2(1 - N_\psi) \\
\sigma_3^N &= \widehat{\sigma_3^N} - \lambda(\alpha_2 - \alpha_1 N_\psi)
\end{aligned}
\tag{1.69}
$$

with

$$
\lambda = \frac{\widehat{\sigma_1^N} - \widehat{\sigma_3^N} N_\phi + 2C\sqrt{N_\phi}}{\left(\alpha_1 - \alpha_2 N_\psi\right) - \left(\alpha_2 - \alpha_1 N_\psi\right) N_\phi}
\tag{1.70}
$$

and, for tensile yielding:

$$
\begin{aligned}
\sigma_1^N &= \widehat{\sigma_1^N} + \lambda\alpha_2 \\
\sigma_2^N &= \widehat{\sigma_2^N} + \lambda\alpha_2 \\
\sigma_3^N &= \widehat{\sigma_3^N} + \lambda\alpha_1
\end{aligned}
\tag{1.71}
$$

with

$$\lambda = \frac{\sigma^t - \widehat{\sigma_3^N}}{\alpha_1} \tag{1.72}$$

Finally, new global stress components are calculated, assuming that the principal directions have not been affected by the occurrence of plastic flow.

The square root of the second invariant, and the modulus of the first invariant of incremental plastic-strain tensor, are used as incremental contributions to measure the amount of plastic strain associated with shear and tensile failure, respectively (see Section 1.6.4.4 in **Constitutive Models**).

By default, both Maxwell and Kelvin viscosity properties, η^M and η^K, are infinite (although stored as zero in *UDEC*'s property arrays). Note that if the default value for η^K is adopted, then the model assumes that $G^K = 0$, even if a different value has been assigned to that property. The default value for G^K is zero, and the default value for G^M is 10^{-20}, irrespective of the system of units adopted.

The default value for the timestep is zero, in which case the program treats the material as elasto-plastic, with only the elastic part of the Maxwell cell active.

If stresses are changed in a *UDEC* model with the **block insitu** command, the internal Kelvin strains, e^K_{ij}, will not be compatible with them, and movement will occur until the strains adjust. To avoid this incompatibility, the internal strains may be set to reflect the current values of stresses. The internal Kelvin strains, e^K_{ij}, are available for user inspection and modification, as **block zone property** variables **strain-kelvin-xx**, **strain-kelvin-eyy**, and **strain-kelvin-exy**. An example *FISH* function to perform this step is given in Example 1.1. This function should be invoked immediately following initialization of stresses.

Example 1.1 Function to set Kelvin strains according to stresses

```
;file: kstrain.fis
def setKstrains
  iab = block.head
  loop while iab # 0
    iaz = bl.zone(iab)
    loop while iaz # 0
      kg2 = 2.0 * bl.zone.prop(iaz,'shear_kelvin')
      if kg2 > 0.0 then
        sig0 = (bl.zo.str.xx(iaz) + bl.zo.str.yy(iaz) ...
               + bl.zo.str.zz(iaz))/3.0
        k_exx = (bl.zo.str.xx(iaz) - sig0) / kg2 ;deviatoric stresses
        k_eyy = (bl.zo.str.yy(iaz) - sig0) / kg2
        k_ezz = (bl.zo.str.zz(iaz) - sig0) / kg2
        k_exy =  bl.zo.str.xy(iaz) / kg2
      endif
      iaz = bl.zone.next(iaz)
    end_loop
    iab = bl.next(iab)
  end_loop
end
```

1.2.6 The Power-Law Viscoplastic Model

The viscoplastic model CPOW combines the behavior of the viscoelastic two-component Norton power law and the Mohr-Coulomb elasto-plastic models. In the model formulation, the total strain rate, $\dot{\epsilon_{ij}}$, is decomposed into elastic ($\dot{\epsilon}^e_{ij}$), viscous ($\dot{\epsilon}^c_{ij}$) and plastic ($\dot{\epsilon}^p_{ij}$) components:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}^e_{ij} + \dot{\epsilon}^c_{ij} + \dot{\epsilon}^p_{ij} \tag{1.73}$$

The elastic strain rate, $\dot{\epsilon}^e_{ij}$, is the only component contributing to the stress rate; the deviatoric behavior is visco-elasto-plastic, and is expressed as

$$\dot{S}_{ij} = 2G\left(\dot{e}_{ij} - \dot{e}^c_{ij} - \dot{e}^p_{ij}\right) \tag{1.74}$$

where $\dot{S}_{ij}$ and $\dot{e}_{ij}$ are the deviatoric parts of the stress and strain rate tensors, $\dot{\sigma}_{ij}$ and $\dot{\epsilon}_{ij}$, and G is tangent shear modulus (see Section 1.2.5 for a definition of the notation convention used in this section).

The volumetric behavior is elasto-plastic, and has the form

$$\dot{\sigma}_0 = K(\dot{e}^e_{vol} - \dot{e}^p_{vol}) \tag{1.75}$$

where $\dot{\sigma}_0 = (\dot{\sigma}_{11} + \dot{\sigma}_{22} + \dot{\sigma}_{33})/3$, $\dot{e}_{vol} = \dot{e}_{11} + \dot{e}_{22} + \dot{e}_{33}$, and K is tangent bulk modulus.

Creep is activated by the von Mises stress $q = \sqrt{3J_2}$ in accordance with Norton power law ($J_2 = 1/2 S_{ij} S_{ij}$ is the second invariant of stress deviator tensor), and the creep rate is

$$\dot{e}^c_{ij} = \dot{e}_{cr} \frac{\partial q}{\partial S_{ij}} \tag{1.76}$$

The direction of creep flow is derived from the definition of q:

$$\frac{\partial q}{\partial S_{ij}} = \frac{3}{2} \frac{S_{ij}}{q} \tag{1.77}$$

By definition, the creep intensity has two components (see Section 1.2.3):

$$\dot{e}_{cr} = \dot{e}^1_{cr} + \dot{e}^2_{cr} \tag{1.78}$$

where

$$\dot{e}^1_{cr} = \begin{cases} A_1\, q^{n_1} & q \geq \sigma_1^{ref} \\ 0 & q < \sigma_1^{ref} \end{cases}$$

$$\dot{e}^2_{cr} = \begin{cases} A_2\, q^{n_2} & q \leq \sigma_2^{ref} \\ 0 & q > \sigma_2^{ref} \end{cases}$$

and σ_1^{ref} and σ_2^{ref} are two model parameters.

The plastic strain rate is defined using the Mohr-Coulomb flow rule:

$$\dot{e}^p_{ij} = \dot{e}_p \frac{\partial g}{\partial \sigma_{ij}} - \frac{1}{3} \dot{e}^p_{vol} \delta_{ij} \tag{1.79}$$

where

$$\dot{e}^p_{vol} = \dot{e}_p \left[\frac{\partial g}{\partial \sigma_{11}} + \frac{\partial g}{\partial \sigma_{22}} + \frac{\partial g}{\partial \sigma_{33}} \right] \tag{1.80}$$

The direction of plastic flow $\partial g/\partial\sigma_{ij}$ is expressed using the definition of the Mohr-Coulomb potential function, g, and the plastic flow rate intensity, $\dot{e}_p$, is derived from the Mohr-Coulomb yield criterion, $f = 0$ (see Section 1.3.2 in **Constitutive Models**). In the principal axes formulation, the yield and potential functions for shear yielding are

$$f = \sigma_1 - \sigma_3 N_\phi + 2C\sqrt{N_\phi} \tag{1.81}$$

$$g = \sigma_1 - \sigma_3 N_\psi \tag{1.82}$$

and for tension yielding, the functions are

$$f = \sigma^t - \sigma_3 \tag{1.83}$$

$$g = -\sigma_3 \tag{1.84}$$

where σ_1 and σ_3 are the minimum and maximum principal stresses (compression negative), C is the material cohesion, ϕ the friction, ψ is the material dilation, σ^t is the tensile strength, $N_\phi = (1+\sin\phi)/(1-\sin\phi)$, and $N_\psi = (1+\sin\psi)/(1-\sin\psi)$.

The model implementation closely follows the procedures described in the manual for the power law and Mohr-Coulomb models. First, the viscoelastic response is calculated for the timestep, Δt. Principal stresses and principal directions are computed, and the yield criterion is checked. If the criterion is not met, plastic strain increments are added for the step, and the increment intensity, $\lambda = \dot{e}_p \Delta t$, is calculated in order to fulfill the yield condition, $f = 0$. The procedure follows the lines presented in the Mohr-Coulomb model implementation, with the viscoelastic response replacing the "elastic guess" for the step.

1.2.7 The WIPP-Creep Viscoplastic Model

Viscoplasticity is also modeled by combining the viscoelastic WIPP model with the Drucker-Prager plasticity model. Of the plasticity models currently embodied in *UDEC*, the Drucker-Prager model is the most compatible with the WIPP-reference creep law, because both models are formulated in terms of the second invariant of the deviatoric stress tensor. Viewed in the pi-plane, both models exhibit responses that depend only on radial distance from the isotropic-stress locus. The response of the Mohr-Coulomb model, on the other hand, is not isotropic because the intermediate principal stress does not enter into its formulation.

The following development is slightly different from that presented in Section 1.3.2 in **Constitutive Models**, and is provided to demonstrate the compatibility of the Drucker-Prager formulation with the creep formulation given in Section 1.2.4.

The shear yield function for the Drucker-Prager model is (see Eq. (1.37) in **Constitutive Models**)

$$f^s = \tau + q_\phi \sigma_\circ - k_\phi \tag{1.85}$$

where $f^s = 0$ at yield, $\sigma_\circ = \sigma_{kk}/3$, and $\tau = \sqrt{J_2}$, where J_2 is the second invariant of the deviatoric stress tensor. Parameters q_ϕ and k_ϕ are material properties.

Because

$$J_2 = \sigma^d_{ij}\sigma^d_{ij}/2 \tag{1.86}$$

(e.g., see Malvern 1969, p. 93), τ may be related to the stress magnitude, $\bar{\sigma}$, given in Eq. (1.36):

$$\bar{\sigma} = \sqrt{3}\,\tau \tag{1.87}$$

The plastic potential function in shear, g^s, is similar to the yield function, with the substitution of q_ψ for q_ϕ as a material property that controls dilation (see Eq. (1.40) in **Constitutive Models**):

$$g^s = \tau + q_\psi \sigma_\circ \tag{1.88}$$

If the yield condition ($f^s = 0$) is met, the following flow rules apply

$$\dot{\epsilon}^{dp}_{ij} = \lambda \frac{\partial g^s}{\partial \sigma^d_{ij}} \tag{1.89}$$

$$\dot{\epsilon}^{p}_{\circ} = \lambda \frac{\partial g^s}{\partial \sigma_\circ} \tag{1.90}$$

where λ is a multiplier (not a material property) to be determined from the requirement that the final stress tensor must satisfy the yield condition. Superscript ^{p} denotes “plastic,” and ^{d} denotes “deviatoric.” By differentiating Eqs. (1.31), (1.86) and (1.90), we obtain

$$\dot{\epsilon}^{dp}_{ij} = \lambda \frac{\sigma^d_{ij}}{2\tau} \tag{1.91}$$

$$\dot{\epsilon}^{p}_{\circ} = \lambda q_\psi \tag{1.92}$$

In *UDEC*'s elastic/plastic formulation, these equations are solved simultaneously with the condition $f^s = 0$, and the condition that the sum of elastic and plastic strain-rates must equal the applied strain-rate.

UDEC's Drucker-Prager model also contains a tensile yield surface, with a composite decision function used near the intersection of the shear and tensile yield functions. The tensile yield surface is

$$f^t = \sigma_\circ - \sigma^t \tag{1.93}$$

where σ^t is the tensile yield strength. The associated plastic potential function is

$$g^t = \sigma_\circ \tag{1.94}$$

Using an approach similar to that for shear yield, the strain rates for tensile yield are

$$\dot{\epsilon}_{ij}^{dp} = 0 \tag{1.95}$$

$$\dot{\epsilon}_\circ^{p} = \lambda \tag{1.96}$$

where λ is determined from the condition that $f^t = 0$. Note that the tensile strength cannot be greater than the value of mean stress at which f^s becomes zero (i.e., $\sigma^t < k_\phi / q_\phi$).

When both creep and plastic flow occur, we assume that the associated strain rates act "in series" – i.e.,

$$\dot{\epsilon}_{ij}^{d} = \dot{\epsilon}_{ij}^{de} + \dot{\epsilon}_{ij}^{dv} + \dot{\epsilon}_{ij}^{dp} \tag{1.97}$$

where the terms represent elastic, viscous and plastic strain-rates, respectively. We first treat the case of shear yield, $f^s > 0$. Combining Eqs. (1.30), (1.32) and (1.91):

$$\dot{\epsilon}_{ij}^{d} = \frac{\dot{\sigma}_{ij}^{d}}{2G} + \frac{\sigma_{ij}^{d}}{2\bar{\sigma}}\left\{3\dot{\epsilon} + \sqrt{3}\lambda\right\} \tag{1.98}$$

In contrast to the creep-only model, the volumetric response of the viscoplastic model is not uncoupled from the deviatoric behavior unless $q_\psi = 0$. Combining Eqs. (1.37) and (1.90):

$$\dot{\epsilon}_{kk} \quad = \quad 3\dot{\epsilon}_\circ \quad = \quad \frac{\dot{\sigma}_{kk}}{3K} + \lambda q_\psi \tag{1.99}$$

The iteration procedure embodied in the creep solution scheme can be extended to include plastic strain increments. Eq. (1.38) becomes

$$\sigma_{ij}^{d'} = \sigma_{ij}^{d\circ} + 2G\Delta t\left\{\dot{\epsilon}_{ij}^{d} - \frac{\sigma_{ij}^{d}}{2\bar{\sigma}}\left(3\dot{\epsilon} + \sqrt{3}\lambda\right)\right\} \tag{1.100}$$

And Eq. (1.99) becomes

$$\sigma_{\circ}' = \sigma_{\circ}^{\circ} + (\dot{\epsilon}_{kk} - \lambda q_{\psi})K\Delta t \tag{1.101}$$

The value of λ can be adjusted in each iteration so that the solution converges to $f^s = 0$. Using Newton's method for roots,

$$\lambda' = \lambda^{\circ} - \frac{f^s}{(\frac{\partial f^s}{\partial \lambda})} \tag{1.102}$$

Note that f^s is evaluated with "new" stress components, $\sigma_{ij}^{d'}$. The derivative in Eq. (1.102) can be evaluated as

$$\frac{\partial f^s}{\partial \lambda} = \frac{\partial f^s}{\partial \sigma_{ij}^{d'}}\frac{\partial \sigma_{ij}^{d'}}{\partial \lambda} + \frac{\partial f^s}{\partial \sigma_{\circ}'}\frac{\partial \sigma_{\circ}'}{\partial \lambda} \tag{1.103}$$

Hence,

$$\frac{\partial f^s}{\partial \lambda} = -G\Delta t - K q_{\phi} q_{\psi}\Delta t \tag{1.104}$$

assuming that the mean stress components (σ_{ij}^{d} and $\bar{\sigma}$) are constant.

For tensile yield, $\sigma_{\circ} > \sigma^t$. Further, if the shear stress is nonzero, the following function is used to decide if shear or tensile yield is occurring.

$$h = \tau - \tau_p - \alpha_p(\sigma_{\circ} - \sigma^t) \tag{1.105}$$

where

$$\tau_p = k_{\phi} - q_{\phi}\sigma^t \tag{1.106}$$

$$\alpha_p = \sqrt{1 + q_\phi{}^2} - q_\phi \tag{1.107}$$

Tensile yield is declared if $h < 0$; otherwise, shear yield occurs. In the former case, the last (plastic) term of Eq. (1.97) is zero, and the value of λ is such that Eq. (1.101) reduces to

$$\sigma'_\circ = \sigma^t \tag{1.108}$$

In order to include softening behavior, an accumulated plastic strain, ϵ^{dp}, is computed, based on the second invariant of the deviatoric strain-increment tensor:

$$\epsilon^{dp} := \epsilon^{dp} + \Delta t \sqrt{\dot\epsilon_{ij}^{dp} \dot\epsilon_{ij}^{dp} / 2} \tag{1.109}$$

There is no built-in support for softening tables, but a *FISH* function that scans the grid every few steps, and recomputes properties based on the current value of ϵ^{dp}, may be written.

1.2.8 A Crushed-Salt Constitutive Model

A crushed-salt constitutive model is implemented to simulate volumetric and deviatoric creep compaction behaviors. The model is a variation of the WIPP-reference creep law, and is based on the model described by Sjaardema and Krieg (1987), with an added deviatoric component as proposed by Callahan and DeVries (1991).

1.2.8.1 Definitions

In the crushed-salt constitutive model, the material density, ρ, is a variable that evolves as a function of compressive volumetric strain, ϵ_v, from the initial crushed-salt emplacement value, ρ_i, to the ultimate intact salt density, ρ_f. The relation between rate of change of volumetric strain and density for use in the *UDEC* incremental Lagrangian formulation may be outlined as follows. (Remember that, as a convention, stresses and strains are negative in compression.)

Consider a given material domain of mass, m, which at time, t, has volume, $V_\circ$, and density, $\rho_\circ$, and let the volumetric strain increment, $\Delta\epsilon_v$, correspond to a change in volume, ΔV, and in density from $\rho_\circ$ to ρ during the time interval, Δt. By virtue of mass conservation, we have

$$\rho_\circ V_\circ = \rho(V_\circ + \Delta V) \tag{1.110}$$

and, by definition of volumetric strain, we obtain

$$\rho = \frac{\rho_\circ}{1 + \Delta\epsilon_v} \tag{1.111}$$

Also, from a continuum approach, we may write

$$\rho = \frac{m}{V} \tag{1.112}$$

and the rate-of-change of density of the given mass is

$$\dot{\rho} = -\frac{m}{V^2}\dot{V} \tag{1.113}$$

Using that $\dot{\epsilon_v} = \dot{V}/V$, together with definition Eq. (1.112), we obtain, after some manipulation,

$$\dot{\epsilon_v} = -\frac{\dot{\rho}}{\rho} \tag{1.114}$$

A measure of the crushed-salt compaction is given by the fractional density, F_d, defined as the ratio between actual and ultimate salt densities:

$$F_d = \frac{\rho}{\rho_f} \tag{1.115}$$

In the model implementation, it is assumed that the creep-compaction mechanism is irreversible (the density can only increase, and cannot decrease) and bounded (no further compaction occurs after the intact salt value has been reached).

1.2.8.2 Constitutive Equations

In the crushed-salt model, elastic stress- and strain-rates are related by means of the incremental expression of Hooke's law:

$$\dot{\sigma}_{ij} = 2G\left[\dot{\epsilon}^e_{ij} - \frac{\dot{\epsilon}^e_{kk}}{3}\delta_{ij}\right] + K\dot{\epsilon}^e_{kk}\delta_{ij} \tag{1.116}$$

where δ_{ij} is the Kronecker delta.

In this expression, the bulk modulus, K, and shear modulus, G, are related to the density by a nonlinear empirical law of the form

$$K = K_f e^{K_1(\rho-\rho_f)} \tag{1.117}$$

$$G = G_f e^{G_1(\rho-\rho_f)} \tag{1.118}$$

where ρ_f, K_f and G_f are properties of the intact salt, and K_1, G_1 are two constants determined from the condition that bulk and shear must take their initial values at the initial value of the density.

It is assumed that, for density values below that of the intact salt, the total strain-rate, $\dot{\epsilon}_{ij}$, can be expressed as the sum of three contributions: nonlinear elastic, $\dot{\epsilon}^e_{ij}$; viscous compaction, $\dot{\epsilon}^c_{ij}$; and viscous shear, $\dot{\epsilon}^v_{ij}$. The elastic strain-rate takes the form

$$\dot{\epsilon}^e_{ij} = \dot{\epsilon}_{ij} - \dot{\epsilon}^c_{ij} - \dot{\epsilon}^v_{ij} \tag{1.119}$$

The viscous compaction term is based on an experimental compaction-rate law of the form

$$\dot{\rho}^c = -B_0\left[1 - e^{-B_1\sigma}\right]e^{B_2\rho} \tag{1.120}$$

where $\sigma = \sigma_{kk}/3$ is the mean stress, and B_0, B_1, B_2 are constants determined experimentally from results of isotropic compaction tests.

The volumetric compaction strain-rate, $\dot{\epsilon}^c_v$, may be derived after substitution of the expression Eq. (1.120) for $\dot{\rho}$ in Eq. (1.114):

$$\dot{\epsilon}^c_v = \frac{1}{\rho}B_0\left[1 - e^{-B_1\sigma}\right]e^{B_2\rho} \tag{1.121}$$

In the *UDEC* implementation, it is assumed that volumetric compaction can only take place if the mean stress is compressive. Furthermore, a cap is assumed for the above expression so that no further compaction arises once the intact salt density has been reached.

1.2.8.3 Viscous Compaction

The total compaction strain-rate has the expression

$$\dot{\epsilon}_{ij}^{c} = \dot{\epsilon}_{v}^{c}\left[\frac{\delta_{ij}}{3} - \beta\frac{\sigma_{ik}^{d}\delta_{kj}}{\bar{\sigma}}\right] \tag{1.122}$$

where σ_{ij}^{d} is the deviatoric stress tensor, $\bar{\sigma} = \sqrt{3J_2}$, and $J_2 = \sigma_{ij}^{d}\sigma_{ij}^{d}/2$ (see Eq. (1.86)). In this formula, the parameter β is a constant set equal to one, so that in a uniaxial compression test, the lateral compaction strain-rate components vanish.

1.2.8.4 Viscous Shear

The viscous shear strain-rate corresponds to that of the WIPP-reference creep law (see Eq. (1.32)). The primary creep strain-rate is the same as that given in Eq. (1.34), but the secondary creep strain-rate (Eq. (1.35)) has the deviatoric stress magnitude, $\bar{\sigma}$, divided by the fractional density (Eq. (1.14)). It has the form

$$\dot{\epsilon}_{s} = D\left(\frac{\bar{\sigma}}{F_d}\right)^{n} e^{(-Q/RT)} \tag{1.123}$$

where the parameters are as previously defined.

As the material approaches full compaction, the fractional density approaches one. Because a cap is introduced to eliminate further creep compaction when the intact salt density is reached, the viscous shear behavior evolves toward that of the intact salt. Note that in the framework of the WIPP model, the intact salt creep behavior is triggered by deviatoric stresses, while the volumetric behavior is elastic.

1.2.8.5 Implementation

In the *UDEC* implementation of the crushed-salt model, the total stresses and the strain rates are decomposed into volumetric and deviatoric components. The incremental equations governing the volumetric behavior are linearized and solved explicitly for the mean stress increment. The creep compaction strain-rate is then derived and used in the expression for the deviatoric behavior whose implementation otherwise closely follows that adopted for the WIPP model. Finally, total stresses for the step are evaluated from the updated volumetric and deviatoric components.

1.3 Solving Creep Problems with *UDEC*

1.3.1 Introduction

The major difference between creep and other constitutive models is the concept of problem time in the simulation. For creep runs, the problem time and timestep represent real time, while for static analyses (in the other constitutive models), the timestep is an artificial quantity, used only as a means of stepping to a steady-state condition.

1.3.2 Creep Timestep in UDEC

For time-dependent phenomena such as creep, *UDEC* allows the user to define a timestep. The default for this timestep is zero. In this case, the program treats the material as linearly elastic (viscoelastic models) or elasto-plastic (viscoplastic models), as appropriate. (The command **block creep timestep fix 0** can also be used to stop the creep calculations.) This can be used to attain equilibrium before starting a creep simulation. The constitutive laws for creep make use of the timestep in their equations, so timestep may affect the response.

Although the user may set the timestep, it is not arbitrary. If it is desired that a system always be in mechanical equilibrium (as in a creep simulation), the time-dependent stress changes produced by the constitutive law must not be large compared to the strain-dependent stress changes. Otherwise, out-of-balance forces will be large, and inertial effects (which are theoretically absent) may affect the solution.

The creep processes are governed by the deviatoric stress state. An estimate for the maximum creep timestep for numerical accuracy can be expressed as the ratio of the material viscosity to the shear modulus:

$$\Delta t^{cr}_{max} = \frac{\eta}{G} \tag{1.124}$$

For the power law, the viscosity may be estimated as the ratio of the stress magnitude, $\bar{\sigma}$, to the creep rate, $\dot{\epsilon}_{cr}$. Using Eq. (1.20), the maximum creep timestep is

$$\Delta t^{cr}_{max} = \frac{\bar{\sigma}^{1-n}}{AG} \tag{1.125}$$

For the WIPP model, the viscosity may be estimated as the ratio of $\bar{\sigma}$ to the secondary creep rate, $\dot{\epsilon}_s$, and using Eq. (1.35), the maximum creep timestep is

$$\Delta t^{cr}_{max} = \frac{e^{Q/RT}}{G\ D\ \bar{\sigma}^{n-1}} \tag{1.126}$$

For the Burgers-creep viscoplastic model, Eq. (1.124) must be interpreted as

$$\Delta t^{cr}_{\max} = \min\left(\frac{\eta^K}{G^K}, \frac{\eta^M}{G^M}\right) \tag{1.127}$$

where the superscripts $.^K$ and $.^M$ refer to Kelvin and Maxwell properties, respectively.

The timestep limitation for creep compaction involves the volumetric response of the system, and is estimated as the ratio of viscosity to bulk modulus. This viscosity may be expressed as the ratio of $\bar{\sigma}$ to the volumetric creep compaction rate, $\dot{\epsilon}^c_v$. Using Eq. (1.121), the maximum creep timestep for creep compaction is

$$\Delta t^{cr}_{max} = \frac{|\sigma|\rho}{K B_0 \left[e^{B_1|\sigma|} - 1\right] e^{B_2\rho}} \tag{1.128}$$

It is recommended that a creep analysis with *UDEC* begin with an initial creep timestep approximately two to three orders of magnitude smaller than Δt^{cr}_{max}, as calculated from the appropriate formula above. By invoking **block creep timestep automatic**, use can then be made of the automatic timestep adjustment, as described in Section 1.3.3. As a rule, the maximum value for the timestep (**block creep timestep maximum**) should not exceed the value derived for Δt^{cr}_{max}.

The stress magnitude, $\bar{\sigma}$, used in the calculation for Δt^{cr}_{max}, can be determined from the initial stress state before the creep process begins. $\bar{\sigma}$, also known as the von Mises stress invariant, can be calculated from the *FISH* function given in Example 1.2. The maximum $\bar{\sigma}$ in the *UDEC* model should be used to calculate Δt^{cr}_{max}.

Example 1.2 von Mises stress invariant ("MISES.FIS")

```
fish def mises
;file: mises.fis
; --- calculate Von Mises stress ---
  max_mises = 0.0
  iab = block.head
  loop while iab # 0
    p_z = bl.zone(iab)
    loop while p_z # 0
      mstr = (bl.zo.str.xx(p_z) + bl.zo.str.yy(p_z) ...
                + bl.zo.str.zz(p_z)) / 3.
      dsxx = bl.zo.str.xx(p_z) - mstr
      dsyy = bl.zo.str.yy(p_z) - mstr
      dszz = bl.zo.str.zz(p_z) - mstr
      dsxy = bl.zo.str.xy(p_z)
      vmstr2 = 1.5 * (dsxx*dsxx + dsyy*dsyy + dszz*dszz)
      vmstr2 = vmstr2 + 9. * (dsxy*dsxy)
```

```
        if vmstr2 > 0.0 then
          z_mises = math.sqrt(vmstr2)
        else
          z_mises = 0.0
        endif
        max_mises = math.max(max_mises,z_mises)
        p_z = bl.zone.next(p_z)
      end_loop
      iab = bl.next(iab)
    end_loop
  end
  @mises
  fish list @max_mises
```

1.3.3 Automatic Adjustment of the Creep Timestep

The timestep may be set by the user to a constant value, or controlled by *UDEC* to change automatically. If the timestep is changed automatically, it can be decreased whenever the maximum unbalanced force exceeds some threshold, and increased whenever it goes below some other level. The threshold is defined as the *ratio* of the maximum unbalanced force to the average gridpoint force.

Typical out-of-balance force criteria for the problem being solved can be determined by observing the out-of-balance force that occurs near equilibrium in the initial stage of the problem when only elastic effects are present. In many cases, a good performance can be obtained by using a gradual increase or decrease of timestep (e.g., with the default ratios **block creep timestep lower-multiplier** = 2.0 and **block creep timestep upper-multiplier** = 0.5).

In some cases, it may be preferable to avoid a continuous adjustment of the timestep, which may create “noise.” For this purpose, after a timestep change has occurred, there is a user-defined “latency period” (e.g., 100 steps) during which no further adjustments are made, allowing the system to settle. Normally, the timestep will start at a small value, to accommodate transients such as excavation, and then increase as the simulation proceeds. If a new transient is introduced, it may be desirable to reduce the timestep manually, and then let it increase again automatically.

The **block creep timestep** command is used to set the timestep and the parameters required to allow the timestep to change automatically. The keywords are listed in Section 1.4.1.

1.3.4 Temperature Dependency

For the WIPP model, the creep rate is temperature-dependent. Temperatures may be supplied to the WIPP model in one of two ways: they may be specified as a property of the WIPP model; or they may be calculated using the thermal option of the code. With the first approach, temperatures are assigned with the **block zone property temperature** command and do not change during the calculation. In the second approach, the **block config thermal** command must first be given, and temperatures can be initialized with the **block grid init temperature** command. In the latter case, temperatures can change during cycling. A temperature gradient may be specified with both approaches.

1.3.5 Modified Damping Formulation

In the regular damping formulation, the damping force, F_d, is

$$F_d = -\alpha |F| \mathrm{sgn}(\dot{u}) \tag{1.129}$$

and the equation of motion of a gridpoint, in simplified form, is

$$\Delta \dot{u} = F(1 \pm \alpha)\frac{\Delta t}{m} \tag{1.130}$$

where F is the unbalanced force, m is the gridpoint mass and α is the damping factor. The sign of α depends on the sign of velocity and the sign of F. The term $(1 \pm \alpha)$ thus acts as a variable multiplier on m, such that mass is removed at the maximum velocity and added when the velocity passes through zero. In this way, a fraction of the kinetic energy is removed twice per cycle of oscillation.

The scheme described above is efficient at removing kinetic energy when the velocity components of most gridpoints pass through zero periodically, since the mass-adjustment process depends on velocity sign-changes. In creep simulations, it is common for the steady-state solution to involve motion of all gridpoints. The damping effect on the system is zero in these cases, due to the lack of sign-changes in velocity components. The effect can be demonstrated even in an elastic example, when the final state is one of uniform motion of the whole body (see Example 1.3), which models a block under gravity. However, the lower boundary is constrained to move upwards at constant velocity, and all gridpoints are fixed in the horizontal direction. The final "equilibrium" state should be one in which gravity-induced stresses act in the body but, in addition, all gridpoints should move upwards at the same velocity. As seen from the velocity history in Figure 1.2, the body continues to oscillate indefinitely, and does not reach the predicted steady state.

Example 1.3 Elastic block with gravity and imposed velocity at lower boundary

```
model new
;file creep_03.dat
```

```
; Local Damping (default)
model title 'Local Damping'
block largestrain off
block tolerance corner-round-length .001
block create polygon 0,0 0,5 5,5 5,0
block zone gen edge 1
block zone cmodel assign elas
block zone property dens 1 shear 1e3 bu .7e3
block mechanical grav 0,-10
block gridpoint apply vel-x 0
block gridpoint apply vel-y 1 range pos-y -.1 .1
block gridpoint initial vel-y 1
history interval 5
block gridpoint history vel-y  2.5 5
block mechanical history time-total
block cyc 1000
model save 'creep_01a.sav'



;*** BRANCH: COMBINED DAMPING ****
model new
; Combined damping
model title 'Combined Damping'
block largestrain off
block config creep ; automatically switches to combined damping
block tolerance corner-round-length .001
block create polygon 0,0 0,5 5,5 5,0
block zone gen edge 1
block zone cmodel assign elas
block zone property dens 1 shear 1e3 bu .7e3
block mechanical grav 0,-10
block gridpoint apply vel-x 0
block gridpoint apply vel-y 1 range pos-y -.1 .1
block gridpoint initial vel-y 1
history interval 5
block gridpoint history vel-y  2.5 5
block mechanical history time-total
block cyc 1000
model save 'creep_01b.sav'
```

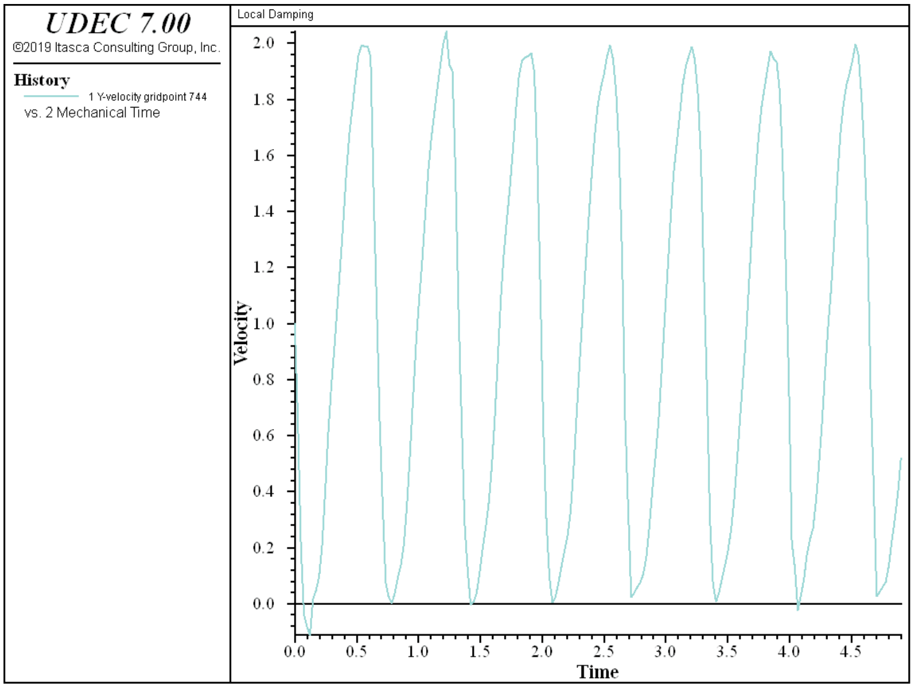

Figure 1.2 y-velocity history at top of block – regular damping

In order to develop a damping formulation that is insensitive to rigid-body motion, consider periodic motion superimposed on steady motion:

$$\dot{u} = V \sin(\omega t) + \dot{u}_\circ \tag{1.131}$$

where V is the maximum periodic velocity, ω is the angular frequency, and $\dot{u}_\circ$ is the superimposed steady velocity. Differentiating twice, and noting that $m\ddot{u} = F$,

$$\dot{F} = -mV\omega^2 \sin(\omega t) \tag{1.132}$$

In Eq. (1.132), $\dot{F}$ is proportional to the periodic part of $\dot{u}$, without the constant $\dot{u}_\circ$. We may substitute $-\mathrm{sgn}(\dot{F})$ in Eq. (1.129) to obtain the same damping force, if the motion is periodic:

$$F_d = \alpha |F| \mathrm{sgn}(\dot{F}) \tag{1.133}$$

This equation is insensitive to a constant offset in velocity, since $\dot{F}$ does not involve $\dot{u}_\circ$. In practice, Eq. (1.131) is not as efficient as Eq. (1.129) if the motion is not strictly periodic. For the creep option of *UDEC*, it is found that the combination of both formulas in equal proportions gives good results:

$$F_d = \alpha |F| \big(\mathrm{sgn}(\dot{F}) - \mathrm{sgn}(\dot{u})\big)/2 \tag{1.134}$$

If we run Example 1.3 in creep mode (inserting **block config creep** before the data set), the system then converges to the steady state in which all velocities are equal to the imposed velocity (see Figure 1.3, for the velocity history of one top-surface gridpoint), and the internal stresses exhibit the same gravitational gradient as the static case.

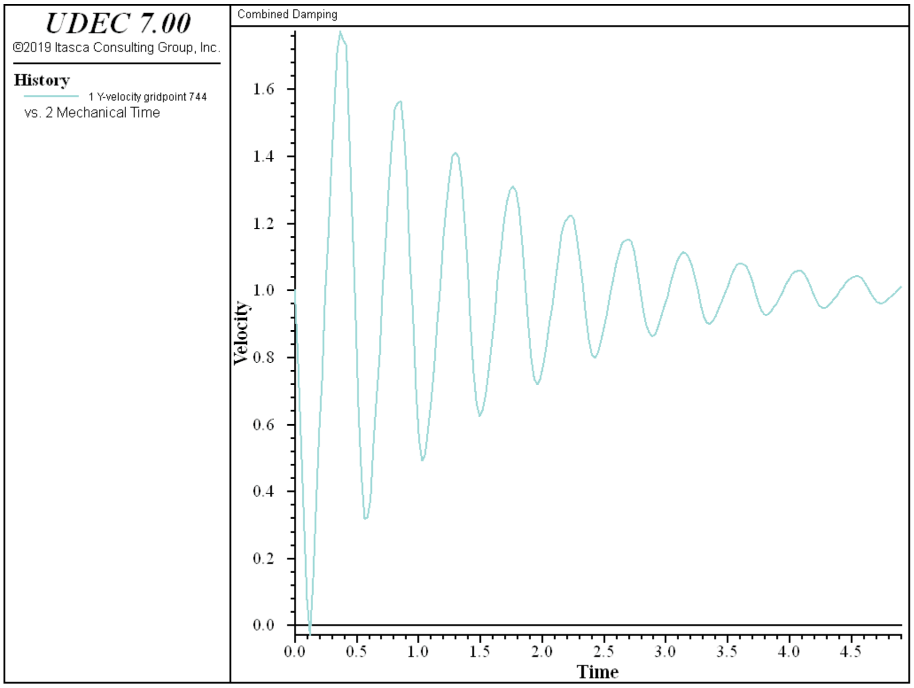

Figure 1.3   y-velocity history at top of block – combined damping

This form of damping is termed *combined* damping, and is also available when creep is not active (using the **block mechanical damping combined** command).

Different forms of damping can be used in creep simulations by using the **block mechanical damping** command. On the choice of damping for a creep calculation, three recommendations are made:

1. Local damping is more appropriate when creep or plastic flow is localized to a small portion of the model (which is usually the case for plasticity for which local damping is the default setting).
2. Combined damping is more effective in most creep runs in which creep flow can affect large portions of the model.
3. When in doubt, it is recommended that displacement histories be monitored in the region of interest to ensure that a monotonic path is followed.

1.4 Input Instructions for Creep Modeling

1.4.1 UDEC Commands

All commands have the same structure as those in the standard version of *UDEC*. No new commands are required, but additional keywords are used with existing commands. The command **block config creep** is required to enable creep modeling. The other keywords for each **block creep** command are described below.

history keyword

The following creep parameter values can be sampled and stored during a model run.

timestep creep timestep

time-total creep time. This allows plotting parameter histories versus creep time.

list keyword <keyword> . . . <**ra**nge . . . >

Printed output is produced according to the keyword(s) below. If a **range** is specified, then printed output will be restricted to the given range.

The following keywords may be used.

creep information parameters for the creep model

time-total *t*

The mechanical time is initialized to ***t***. This is useful if creep is to be started at a time other than zero. The default is ***t*** = 0.

timestep keyword <keyword ***value***> . . .

This command sets timestep parameters for a creep analysis. The following keywords are available.

automatic <**on/off**>

The automatic calculation of the creep timestep is turned on and off with the **automatic on** and **automatic off** keywords. By turning on this option, the timestep will be updated automatically. The automatic timestep calculation is controlled by the **timestep** keywords **lower-bound**, **upper-bound**, **lower-multilplier**, **upper-multiplier** and **latency**. The default is **starting**.

fix *v* <**auto** <**on/off**>>

Defines the static creep timestep.

latency *n*

The minimum number of creep timesteps that must elapse before the timestep is changed is set to ***v***. The default is ***v*** = 100, and this value should be sufficient for most creep analyses.

lower-**b**ound *v*

The creep timestep will be increased if the unbalanced force falls below ***v***. The default is ***v*** = 10^4.

lower-**mul**tiplier *v*

The creep timestep will be multiplied by ***v*** if the unbalanced force falls below **lower-bound**. **lower-multiplier** must be greater than 1. The default is ***v*** = 2.

maximum *v*

The maximum creep timestep allowed is set to ***v***. The default is no limit on creep timestep.

minimum *v*

The minimum creep timestep allowed is set to ***v***. The default is ***v*** = 0.

upper-**b**ound *v*

The creep timestep will be decreased if the unbalanced force exceeds ***v***. The default is ***v*** = 10^5.

upper-**mul**tiplier *v*

The creep timestep will be multiplied by ***v*** if the unbalanced force ratio exceeds **upper-limit**. **upper-multiplier** must be less than 1. The default is ***v*** = 0.5.

Other commands used in creep modeling.

block solve keyword ***value***

This command controls the automatic timestepping for creep calculations. A calculation is performed until the limiting condition, as defined by the following keyword, is reached.

age *t*

In **creep** mode, ***t*** is the mechanical time limit for the creep calculation.

age-off removes a previously specified **age** time limit.

block **zo**ne **cm**odel **ass**ign keyword <**ra**nge . . . >

This command associates a constitutive model with an area of the grid corresponding to a range of zones.

During the calculation, zones will behave according to a creep model corresponding to one of the following keywords.

burgers	Burgers model
burger-**m**ohr	Burgers-creep viscoplastic model
maxwell	classical viscosity
power	two-component creep power law
power-**m**ohr	power-law viscoplastic model
wipp	WIPP-reference creep formulation
wipp-**d**rucker	WIPP-creep viscoplastic model
wipp-**s**alt	crushed-salt model

1.4.2 FISH ***Variable***

The following scalar variables are available to use in a *FISH* function to assist with creep analysis.

block**.cr**eep**.timestep**	creep timestep
block**.cr**eep**.ti**me**.total**	creep time

1.5 Properties for Creep Constitutive Models

1.5.1 Classical Viscoelastic (Maxwell Substance) – **block zone cmodel assign maxwell**

(1)	**bu**lk	elastic bulk modulus, K
(2)	**dens**ity	material density, ρ
(3)	**sh**ear	elastic shear modulus, G
(4)	**vis**cosity	dynamic viscosity, η

1.5.2 *Burgers Model* – **block zone cmodel assign burgers**

(1)	**bul**k	elastic bulk modulus, K
(2)	**dens**ity	material density, ρ
(3)	**sh**ear-**k**elvin	Kelvin shear modulus, G^K
(4)	**sh**ear-**m**axwell	Maxwell shear modulus, G^M
(5)	**visc**osity-**k**elvin	Kelvin viscosity, η^K
(6)	**visc**osity-**m**axwell	Maxwell viscosity, η^M

The following calculated values can be printed or plotted.

(1)	**stra**in-**k**elvin-**xx**	Kelvin strain in x-direction
(2)	**stra**in-**k**elvin-**xy**	Kelvin shear strain
(3)	**stra**in-**k**elvin-**yy**	Kelvin strain in y-direction

1.5.3 *Power Law* – **block zone cmodel assign power**

(1)	**bu**lk	elastic bulk modulus, K
(2)	**cons**tant**-1**	power-law constant, A_1
(3)	**cons**tant**-2**	power-law constant, A_2
(4)	**dens**ity	material density, ρ
(5)	**ex**ponent**-1**	power-law exponent, n_1
(6)	**ex**ponent**-2**	power-law exponent, n_2
(7)	**sh**ear	elastic shear modulus, G
(8)	**stre**ss**-ref**erence**-1**	reference stress, σ_1^{ref}
(9)	**stre**ss**-ref**ernce**-2**	reference stress, σ_2^{ref}

1.5.4 *WIPP Model* – **block zone cmodel assign wipp**

(1)	**act**ivation-**e**nergy	activation energy, Q
(2)	**bu**lk	elastic bulk modulus, K
(3)	**cons**tant-**a**	WIPP model constant, A
(4)	**cons**tant-**b**	WIPP model constant, B
(5)	**cons**tant-**d**	WIPP model constant, D
(6)	**cons**tant-**gas**	gas constant, R
(7)	**cr**eep-**ra**te-**cr**itical	critical steady-state creep rate, $\dot{\epsilon}^*_{ss}$
(8)	**dens**ity	material density, ρ
(9)	**ex**ponent	WIPP model exponent, n
(10)	**sh**ear	elastic shear modulus, G
(11)	**tem**perature	zone temperature, T

The following calculated values can be printed or plotted.

(1)	**cr**eep-**st**rain-**p**rimary	creep strain
(2)	**cr**eep-**ra**te-**p**rimary	creep strain rate

1.5.5 Burgers-Creep Viscoplastic Model – **block zone cmodel assign burger-mohr**

(1)	**bu**lk	elastic bulk modulus, K
(2)	**coh**esion	cohesion, c
(3)	**dens**ity	material density, ρ
(4)	**dil**ation	dilation angle, ψ
(5)	**f**riction	angle of internal friction, ϕ
(6)	**sh**ear-**k**elvin	Kelvin shear modulus, G^K
(7)	**sh**ear-**m**axwell	elastic shear modulus, G^M
(8)	**ten**sion	tension limit, σ^t
(9)	**visc**osity-**k**elvin	Kelvin viscosity, η^K
(10)	**visc**osity-**m**axwell	Maxwell dynamic viscosity, η^M

The following calculated values can be printed or plotted.

(1)	**stra**in-**k**elvin-**xx**	Kelvin strain in x-direction
(2)	**stra**in-**k**elvin-**xy**	Kelvin shear strain
(3)	**stra**in-**k**elvin-**yy**	Kelvin strain in y-direction

1.5.6 Power Law Viscoplastic Model – **block zone cmodel assign power-mohr**

(1)	**bu**lk	elastic bulk modulus, K
(2)	**coh**esion	cohesion, c
(3)	**cons**tant**-1**	power-law constant, A_1
(4)	**cons**tant**-2**	power-law constant, A_2
(5)	**dens**ity	material density, ρ
(6)	**dil**ation	dilation angle, ψ
(7)	**ex**ponent**-1**	power-law exponent, n_1
(8)	**ex**ponent**-2**	power-law exponent, n_2
(9)	**f**riction	angle of internal friction, ϕ
(10)	**sh**ear	elastic shear modulus, G
(11)	**stre**ss**-ref**erence**-1**	reference stress, σ_1^{ref}
(12)	**stre**ss**-ref**erence**-2**	reference stress, σ_2^{ref}
(13)	**t**ension	tension limit, σ^t

1.5.7 WIPP-Creep Viscoplastic Model – block zone cmodel assign wipp-drucker

(1)	**act**ivation-**e**nergy	activation energy, Q
(2)	**bu**lk	elastic bulk modulus, K
(3)	**coh**esion-**d**rucker	material parameter, k_ϕ
(4)	**cons**tant-**a**	WIPP model constant, A
(5)	**cons**tant-**b**	WIPP model constant, B
(6)	**cons**tant-**d**	WIPP model constant, D
(7)	**cons**tant-**gas**	gas constant, R
(8)	**cr**eep-**ra**te-**cr**itical	critical steady-state creep rate, $\dot{\epsilon}^*_{ss}$
(9)	**dens**ity	material density, ρ
(10)	**dil**ation-**d**rucker	material parameter, q_k
(11)	**ex**ponent	WIPP model exponent, n
(12)	**fric**tion-**d**rucker	material parameter, q_ϕ
(13)	**sh**ear	elastic shear modulus, G
(14)	**tem**perature	zone temperature, T
(15)	**ten**sion	tension limit, σ^t

The following calculated values can be printed or plotted.

(1)	**cr**eep-**st**rain-**p**rimary	creep strain
(2)	**cr**eep-**ra**te-**p**rimary	creep strain rate
(3)	**stra**in-**sh**ear-**p**lastic	accumulated plastic shear strain
(4)	**stra**in-**ten**sion-**p**lastic	accumulated plastic tensile strain

1.5.8 Crushed Salt Model – **block zone cmodel assign wipp-salt**

(1)	**act**ivation-**e**nergy	activation energy, Q
(2)	**bu**lk	elastic bulk modulus, K
(3)	**bulk-f**inal	final, intact salt, bulk modulus, K_f
(4)	**cons**tant-**a**	WIPP model constant, A
(5)	**cons**tant-**b**	WIPP model constant, B
(6)	**cons**tant-**d**	WIPP model constant, D
(7)	**cons**tant-**gas**	gas constant, R
(8)	**cr**eep-**ra**te-**cr**itical	critical steady-state creep rate, $\dot{\epsilon}^*_{ss}$
(9)	**com**paction-**0**	creep compaction parameter, B_0
(10)	**com**paction-**1**	creep compaction parameter, B_1
(11)	**com**paction-**2**	creep compaction parameter, B_2
(12)	**dens**ity	material density, ρ
(13)	**dens**ity-**f**inal	final, intact salt, density, ρ_f
(14)	**dens**ity-**s**alt	density, ρ
(15)	**ex**ponent	WIPP model exponent, n
(16)	**sh**ear	elastic shear modulus, G
(17)	**sh**ear-**f**inal	final, intact salt, shear modulus, G_f
(18)	**tem**perature	zone temperature, T

The following calculated properties can be printed or plotted.

(1)	**fra**ctional-**d**ensity	current fractional density, F_d
(2)	**com**paction-**s**hear	creep compaction parameter, G_1
(3)	**com**paction-**b**ulk	creep compaction parameter, K_1

1.6 Verification and Example Problems

Several examples are presented to validate and demonstrate the creep models in *UDEC*. The data files for these examples are contained in the "\ITASCA\UDEC700\Creep" folder.

1.6.1 Parallel-Plate Viscometer – Classical Model

Suppose that a material with viscosity η is steadily squeezed between two parallel plates that are moving at a constant velocity V_0. The two plates have length 2 l and are a distance 2 h apart. The material is prevented from slipping at the plates. The approximate analytical solution, given by Jaeger (1969), is

$$V_x = \frac{3V_o x\ (h^2 - y^2)}{2h^3} \tag{1.135}$$

$$V_y = \frac{V_o y (y^2 - 3h^2)}{2h^3} \tag{1.136}$$

$$\sigma_{xx} = 3\eta V_o \left[\frac{3(h^2 - y^2) + x^2 - l^2}{2h^3} \right] \tag{1.137}$$

$$\sigma_{yy} = 3\eta V_o \left[\frac{y^2 - h^2 + x^2 - l^2}{2h^3} \right] \tag{1.138}$$

$$\sigma_{xy} = -3 \left[\frac{V_o\ \eta\ x\ y}{h^3} \right] \tag{1.139}$$

The problem is illustrated in Figure 1.4.

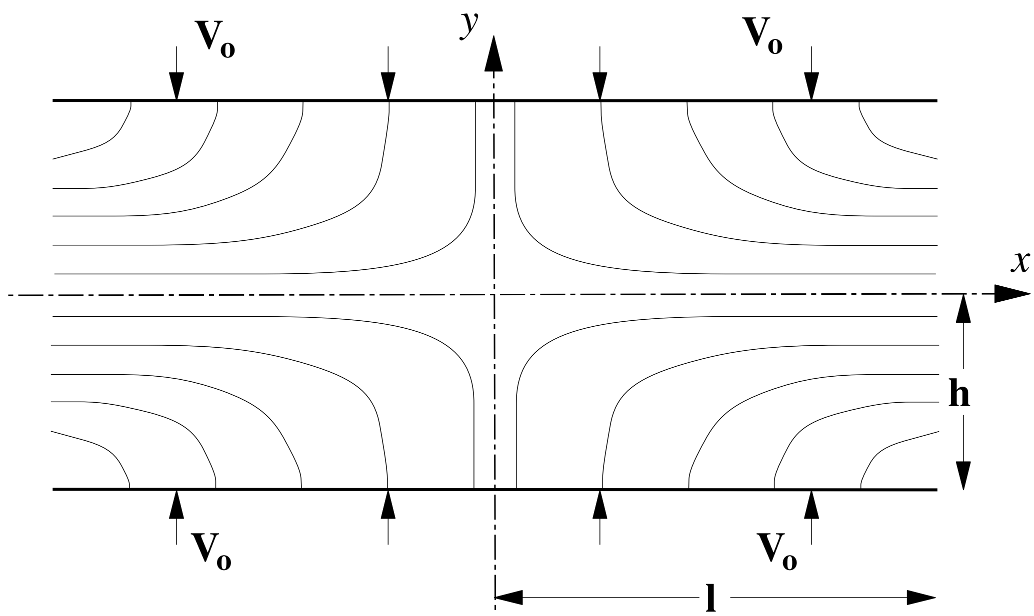

Figure 1.4 Parallel-plate viscometer showing velocity streamlines (Jaeger 1969)

To solve the problem with *UDEC*, advantage can be taken of the symmetry about the x- and y-axes. Only the top-right quadrant needs to be modeled. For compatibility with the approximations of the analytical solution, artificial forces have to be applied at the “free” right-hand edge, and small-strain logic is used.

The material properties are

density	1 kg/m^3
shear modulus	5×10^8 Pa
bulk modulus	1.5×10^9 Pa
viscosity	1×10^{10} kg/ms

The input is given in Example 1.4.

The viscous model component may also be tested with the viscoplastic model (**block zone cmodel assign maxwell**) for the viscometer test. The additional commands for these models are contained in Example 1.4 as comments. Upon execution of the data file with the **maxwell** model activated, results identical to those produced with the classical viscoelastic model are obtained.

Example 1.4 Parallel plate test – classical viscosity

```
model new
;file: creep_04.dat
; classical viscosity - parallel plate test
;
model title 'Parallel-Plate Viscometer (Classical Viscosity)'
block config creep
block largestrain off
block tolerance corner-round-length .001
block create polygon 0 0 0 5 10 5 10 0
block zone gen quad 1.1 1.2
block zone cmodel assign maxwell
block zone property density 1 bu 1.5e9 sh 0.5e9 visc 1e10
bl grid app vel-x   0 range pos-x -.1 .1
bl grid app vel-y   0 range pos-y -.1 .1
bl grid app vel-x   0 range pos-y 4.9 5.1
bl grid app vel-y -18 range pos-y 4.9 5.1
bl grid app force-x 4.5e5                 ran p-x 9.9 10.1 p-y -.1 .1
bl grid app force-x 8.64e5 force-y -2.4e5 ran p-x 9.9 10.1 p-y 0.9 1.1
bl grid app force-x 7.56e5 force-y -4.8e5 ran p-x 9.9 10.1 p-y 1.9 2.1
bl grid app force-x 5.76e5 force-y -7.2e5 ran p-x 9.9 10.1 p-y 2.9 3.1
bl grid app force-x 3.24e5 force-y -9.6e5 ran p-x 9.9 10.1 p-y 3.9 4.1
block creep timestep fix  1
block creep history time
block gridpoint history vel-x  3,3
block gridpoint history vel-y  3,3
block zone history stress-xx 3,3
block zone history stress-yy 3,3
block zone history stress-xy 3,3
block zone history stress-zz 3,3
block step 500
fish def anal
  iab = block.head
  loop while iab # 0
    iaz = bl.zone(iab)
    loop while iaz # 0
      xp = bl.zone.pos.x(iaz)
      yp = bl.zone.pos.y(iaz)
      a_sxx = 3*((height^2)-(yp^2))
      a_sxx = (a_sxx + ((xp^2) - (length^2)))/(2*(height^3))
      a_sxx = a_sxx*3*visc*vel*tdel
      bl.zone.extra(iaz) = -a_sxx
      iaz = bl.zone.next(iaz)
    end_loop
```

```
      iab = bl.next(iab)
    end_loop
  end
  fish set @vel = -18
  fish set @height = 5
  fish set @length = 10
  fish set @visc = 1e10
  @anal
  model save 'creep_04.sav'
  ret
```

Figure 1.5 is a plot of *UDEC*'s resulting σ_{xx} contours.

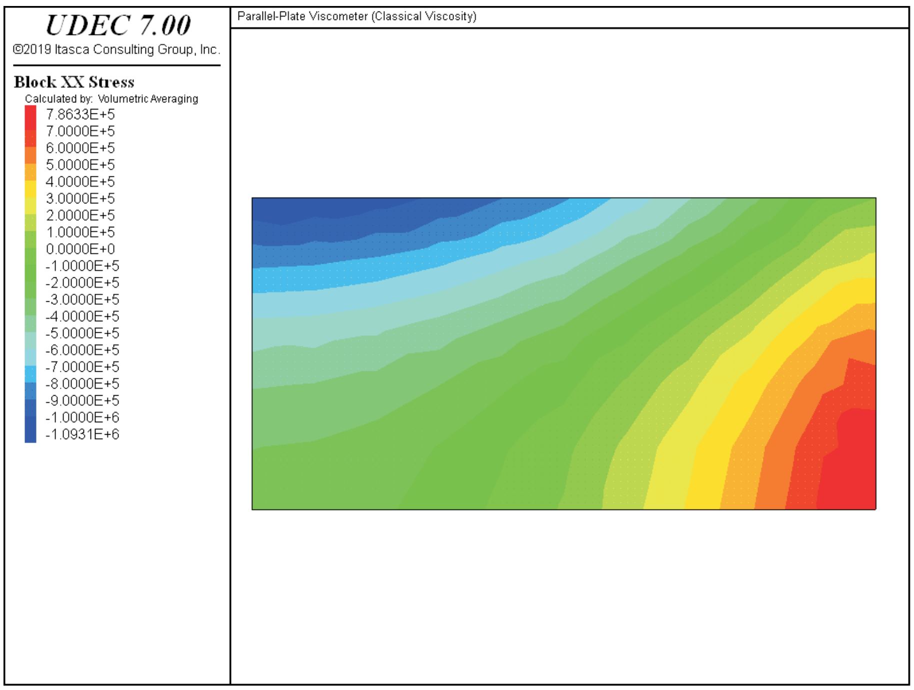

Figure 1.5 ***σ_{xx}-contours for parallel-plate viscometer***

1.6.2 Parallel-Plate Viscometer – WIPP Model

The parallel-plate viscometer test in Section 1.6.1 is repeated to test the WIPP creep model. The analytical solution for the parallel-plate test assumes that the viscosity is constant. In the WIPP model, the viscosity is dependent on the deviatoric stress, so a direct comparison cannot be made.

Example 1.5 contains the commands necessary to run this problem. Note that it is essential to specify the temperature, because it is used by the WIPP creep law. In this case, the **temperature** property is used to input a uniform temperature of 300 K.

The WIPP-model component is also tested in the viscoplastic model (**block zone cmodel assign wipp-drucker**) and the crushed-salt model (**block zone cmodel assign wipp-salt**) for the viscometer test. The values of shear and tensile strength are set high to prevent plastic failure in the viscoplastic model, and the values of initial and final density are set equal to prevent viscous compaction in the crushed-salt model. The additional commands for these models are contained in Example 1.5 as comments. Upon execution of the data file with each model activated, identical results to those produced with the WIPP model are obtained.

The contours of x-velocity using the **wipp** model are shown in Figure 1.6. The same results are produced with the **wipp-drucker** and **wipp-salt** models.

Example 1.5 Parallel plate test – WIPP model

```
model new
;file: creep_05.dat
; WIPP Model - parallel plate test
;
model title 'Parallel-Plate Viscometer (WIPP model)'
block config creep
block largestrain off
block tolerance corner-round-length .001
block create polygon 0 0 0 5 10 5 10 0
block zone gen quad 1.1 1.2
block zone cmodel assign  wipp
block zone property density 2600 bu 20.7e9 sh 12.4e9 temp 300
block zone property constant-gas 1.987 activation-energy 12e3 ...
  exponent 4.9 constant-d 5.79e-36
block zone property constant-a 4.56 constant-b 127 ...
  creep-rate-critical 5.39e-8
;
; WIPP-DRUCKER properties
;block zone cmodel assign wipp-drucker
;block zone property friction-drucker 0.0 dilation-drucker 0.0 ...
;  shear-kelvin 1e10 tension 1e10

; WIPP-SALT properties
;block zone cmodel assign wipp-salt
```

```
;block zone property compaction-0 0.0 compaction-1 0.0 ...
;  compaction-2 0.0 bulk-final 20.7e9 shear-final 12.4e9 ...
;  density-salt 2600 density-final 2600
;
bl grid app vel-x = 0 range pos-x -.1 .1
bl grid app vel-y 0 range pos-y -.1 .1
bl grid app vel-x 0 range pos-y 4.9 5.1
bl grid app vel-y -1e-3 range pos-y 4.9 5.1
bl grid app force-x 4.5e5                 range p-x 9.9 10.1 p-y -.1 .1
bl grid app force-x 8.64e5 force-y -2.4e5 range p-x 9.9 10.1 p-y 0.9 1.1
bl grid app force-x 7.56e5 force-y -4.8e5 range p-x 9.9 10.1 p-y 1.9 2.1
bl grid app force-x 5.76e5 force-y -7.2e5 range p-x 9.9 10.1 p-y 2.9 3.1
bl grid app force-x 3.24e5 force-y -9.6e5 range p-x 9.9 10.1 p-y 3.9 4.1
block creep timestep fix  1e4
block creep history time
block gridpoint history vel-x  3,3
block gridpoint history vel-y  3,3
block zone history stress-xx 3,3
block zone history stress-yy 3,3
block zone history stress-xy 3,3
block zone history stress-zz 3,3
block cycle 900
model save 'creep_05.sav'
ret
```

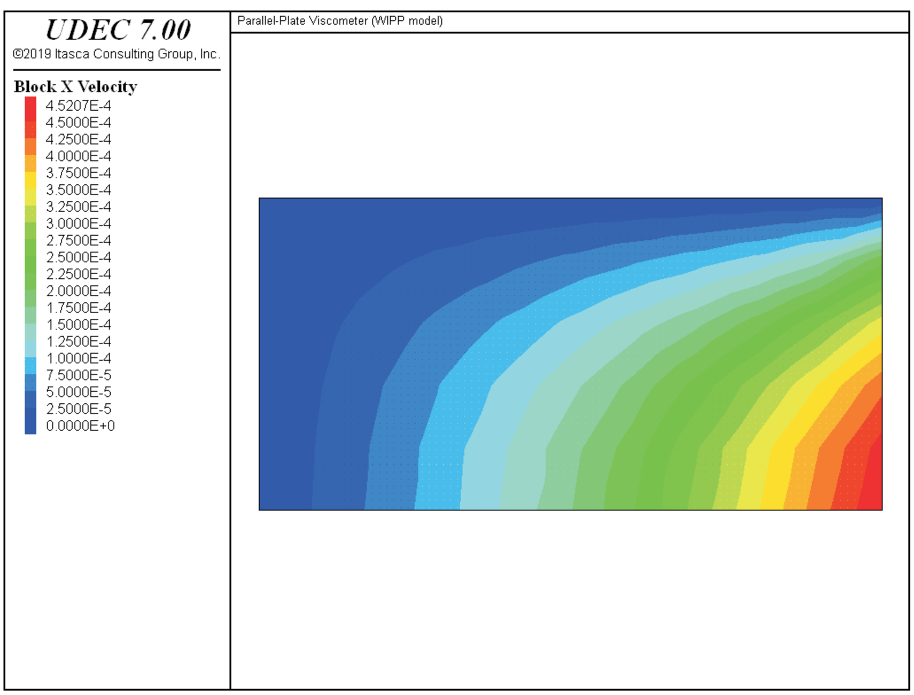

Figure 1.6 Contours of x-velocity from the UDEC WIPP model

1.6.3 Cylindrical Cavity – Power Law

The power law in *UDEC* is used to solve the problem of an infinitely long, thick-walled cylinder. A comparison is made with an analytical solution.

Problem Statement – A cylinder is subject to a pressure on the outer surface. The creep behavior of the material is defined by a single component power law – i.e.,

$$\dot{\epsilon}_{cr} = A\,\bar{\sigma}^n \tag{1.140}$$

For this problem, $A = 1 \times 10^{-7}$ MPa^{-3} yr^{-1} (or 1×10^{-25} Pa^{-3} yr^{-1}), and $n = 3$. The elastic properties of the material are $E = 820$ MPa and $\nu = 0.3636$.

An analytical steady-state solution to this problem has been provided by Van Sambeek (1986) and is reproduced:

$$\sigma_r = -P_b + P_b \left[\frac{(b/r)^{2/n} - 1}{(b/a)^{2/n} - 1} \right] \tag{1.141}$$

$$\sigma_\theta = -P_b - P_b \left[\frac{[(2-n)/n]\,(b/r)^{2/n} + 1}{(b/a)^{2/n} - 1} \right] \tag{1.142}$$

$$\sigma_z = -P_b - P_b \left[\frac{[(1-n)/n]\,(b/r)^{2/n} + 1}{(b/a)^{2/n} - 1} \right] \tag{1.143}$$

$$\dot{u}_r = -A\,(3/4)^{(n+1)/2} \left[P_b\,\frac{2/n}{(b/a)^{2/n} - 1} \right]^n b^2/r \tag{1.144}$$

where σ_r, σ_θ are radial and tangential stress components;

σ_z is the out-of-plane stress component;

P_b is the applied boundary stress;

$\dot{u}_r$ is the radial displacement rate;

a, b are the inner and outer radii of the cylinder, respectively; and

r is the radius to point of calculation.

UDEC Solution – One-quarter of the cylinder was modeled with *UDEC*, as shown in Example 1.6. The outer radius of the cylinder (b) = 20 times the hole radius (a). A pressure of 100 MPa was applied at the outer boundary, the bottom was restrained in the vertical direction, and the left boundary was restrained in the horizontal direction. The last two conditions are required to represent the symmetry correctly.

The initial stresses, corresponding to an elastic cylinder in plane strain without a hole, were

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -P_b = -100 \text{ MPa}$$

The cylinder was allowed to come to elastic equilibrium by setting the creep timestep to zero. Then, the creep timestep was set to its initial value (defined by **block creep timestep minimum**) and allowed to increase automatically until steady state was reached. The maximum allowable timestep (**block creep timestep maximum**) was determined by the procedure described in Section 1.3.2, and using "MISES.FIS" to determine Δt^{cr}_{max}.

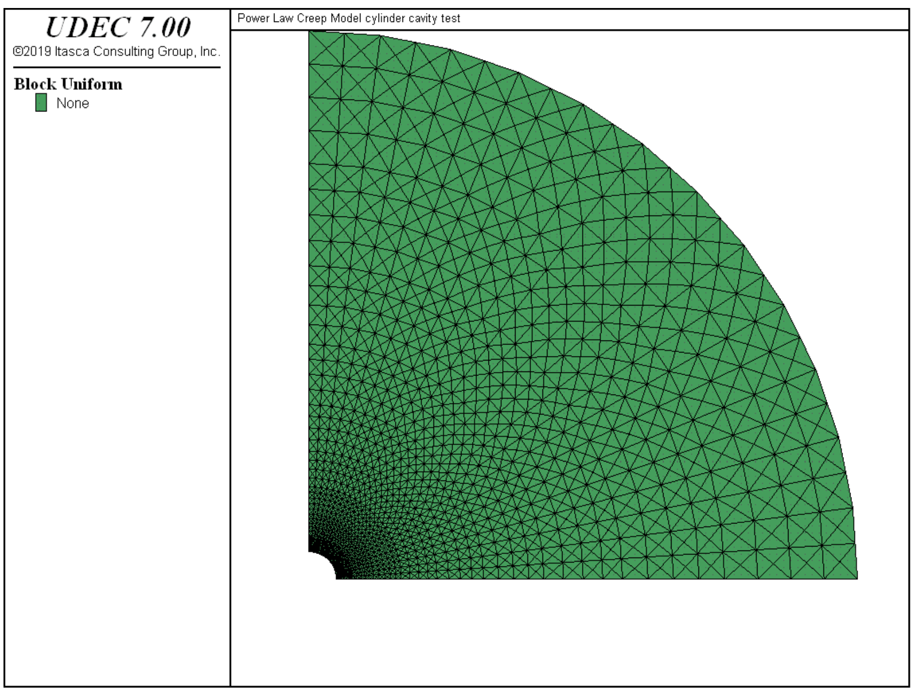

Figure 1.7 UDEC grid for cylindrical cavity test

The data file for this problem is given in Example 1.6. The analytical solution is included in this data file as the *FISH* function **powcyl**.

Example 1.6 Cylindrical cavity – power law

```
model new
;file creep_06.dat
;
; Power Law Creep Model cylinder cavity test
;
model title ...
'Power Law Creep Model cylinder cavity test'
block config creep
block smallstrain

call 'cylinder.fis'
;
block tolerance corner-round-length .001
fish set @a_z = 10.0,
fish set @_a_radius = 1.0,
fish set @r_a = 20.0
@_geometry
@_rings
; use quarter symmetry
;
block delete range pos-x @_mod_size 0 pos-y @_mod_size @mod_size
block delete range pos-x @_mod_size @mod_size pos-y @_mod_size 0
block delete range pos-x 0 @ahalf  pos-y 0 @ahalf
block contact join by-cont
@_gzones

block zone cmodel assign power
block zone property constant-1 = 1e-25, exponent-1 = 3, bu = 1e9 ...
  sh .3e9 dens 2000
block edge apply stress -100e6,0,-100e6 ...
  range ann center 0 0 rad 19.9 20.1
block gridpoint apply vel-x 0 range pos-x -.1 .001
block gridpoint apply vel-y 0 range pos-y -.1 .001
block insitu stress -100e6,0,-100e6 szz -100e6
block gridpoint history vel-x  .5 0
block gridpoint history vel-y  0 .5
block creep timestep fix  0
block solve
;block step 1000
model save 'creep_06a.sav'
;
block gridpoint reset vel
hist reset
hist interval 100
block creep history timestep
```

```
block creep history time-total
block gridpoint history vel-x  .5 0
block gridpoint history vel-y  0 .5
;
; set creep time parameters
;
block creep timestep lower-bound 5e4
block creep timestep upper-bound 5e7
block creep timestep maximum 1.e-4
block creep timestep minimum 1.e-8
block creep timestep start 1e-8
block creep timestep auto
block step 1000000
model sav 'creep_06b.sav'

call 'powcyl.fis'
@compsol
model title 'Radial velocity at hole edge vs time'
model title 'Creep time step'
model title 'Steady-state radial velocity vs distance from hole'
table 11 label 'analytical'
table 1  label 'UDEC'
model title 'Steady-state radial stress vs distance from hole'
table 2 label 'UDEC'
table 12 label 'analytical'
model title 'Steady-state hoop stress vs distance from hole'
table 13 label 'analytical'
table 3 label  'UDEC'
model title 'Steady-state out-of-plane vs distance from hole'
table 4 label 'UDEC'
table 14 label  'analytical'
model sav 'creep_06c.sav'

ret
```

The results for this case are summarized in Figures 1.8 through 1.13. Figure 1.8 shows the radial-velocity (i.e., $\dot{u}_r$) history of a point on the circumference of the hole. Figure 1.9 shows the evolution of the creep timestep, from its initial value of 10^{-7} to the maximum value of $1e^{-4}$. The final steady-state velocity is slightly (1%) below the analytical solution. Figure 1.10 compares the *UDEC* results with the analytical solution for the radial velocity at the steady-state condition. The stress components obtained from *UDEC* are compared with the analytical solution in Figures 1.11 through 1.13. It is obvious from these figures that the *UDEC* results are very close to the analytical solution.

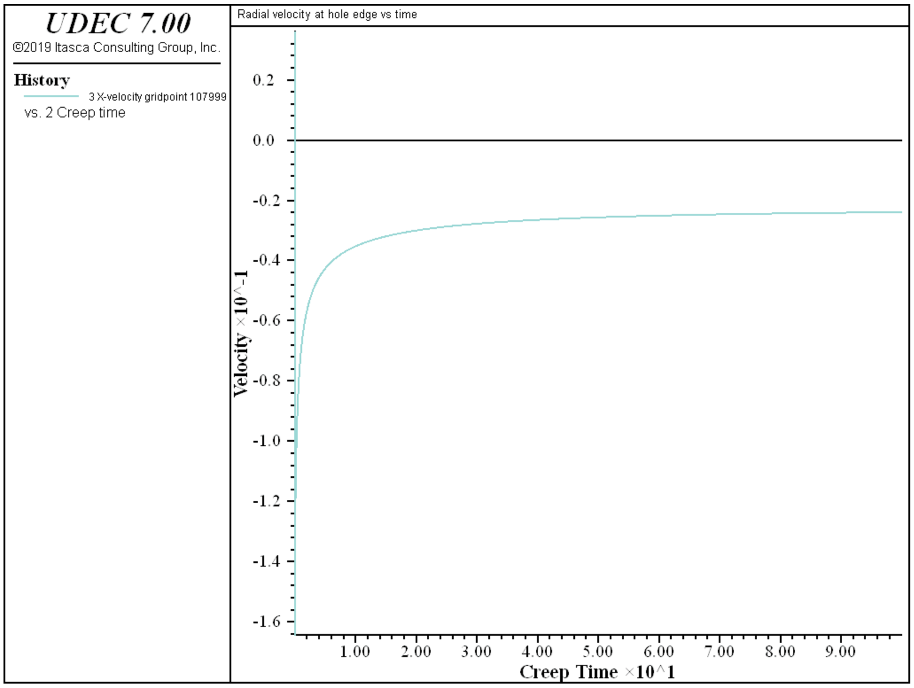

Figure 1.8 Radial velocity at hole edge vs time

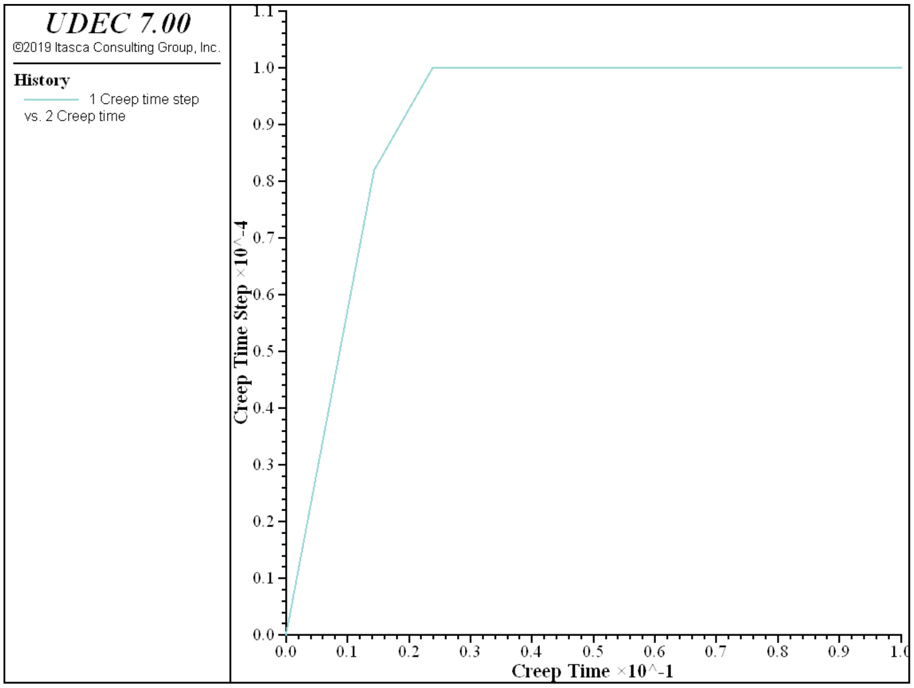

Figure 1.9 History of creep timestep for cylindrical cavity test

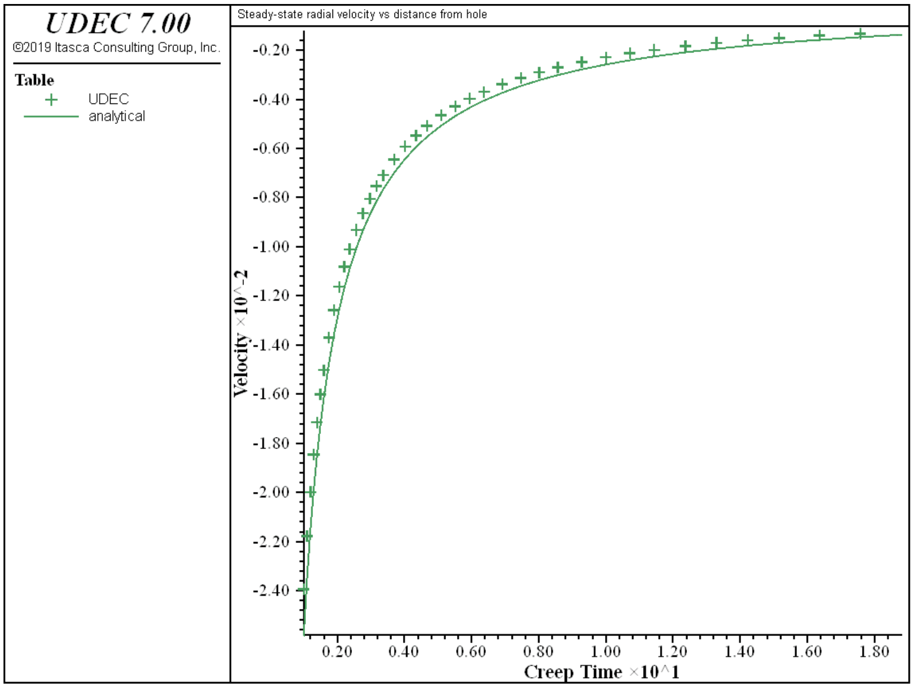

Figure 1.10 Steady-state radial velocity ($\dot{u}_r$) vs distance from hole

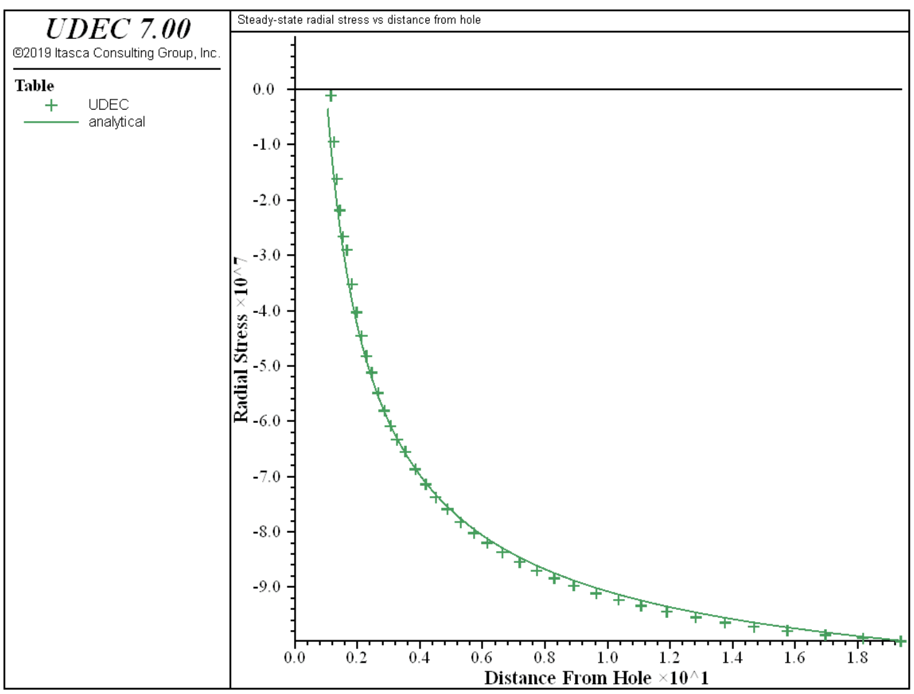

Figure 1.11 Steady-state radial stress (σ_r) vs distance from hole

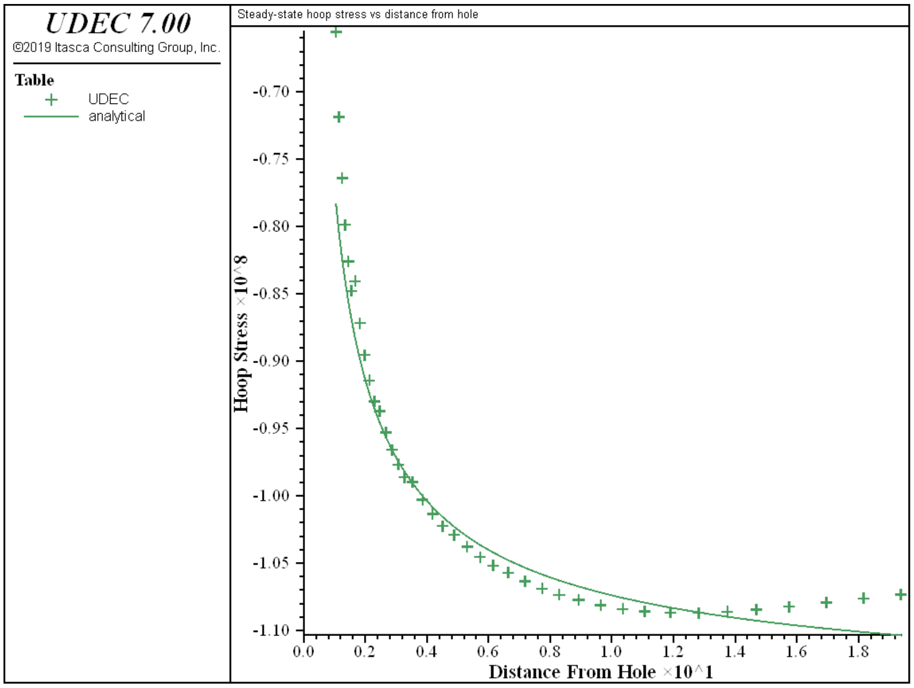

Figure 1.12 Steady-state hoop stress (σ_θ) vs distance from hole

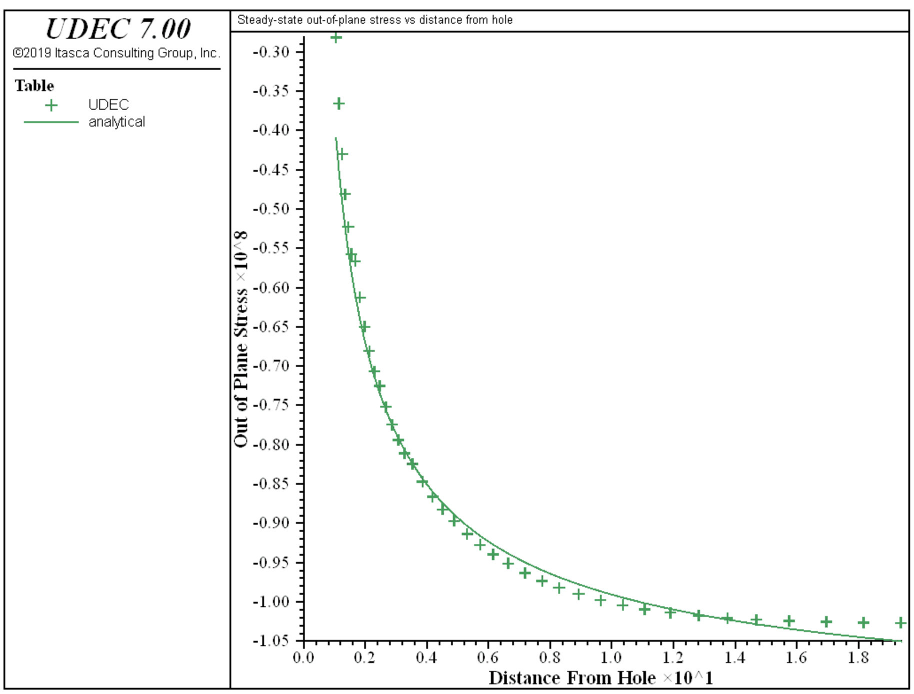

Figure 1.13 Steady-state out-of-plane stress (σ_z) vs distance from hole

1.6.4 Cylindrical Cavity – WIPP Model

The WIPP-reference creep model in *UDEC* is used to solve the problem of radial creep of an infinitely long, thick-walled cylinder subjected to a pressure on its outer surface. The analytical solution, assuming that creep is defined by a single-component power law, is provided in Eqs. (1.141) through (1.144). The WIPP model can be converted to a power law formulation by using only the secondary creep-strain component. This is achieved by setting the WIPP properties **constant-a** and **constant-b** to zero. The WIPP model is then reduced to the form

$$\dot{\epsilon}_{cr} = A\bar{\sigma}^n \tag{1.145}$$

where $A = D\exp(-Q/RT)$.

For this problem, Q = 12,000 cal/mol, R = 1.987 cal/mol K, T = 300° K, $D = 5.5299 \times 10^{-17}$ (or $A = 1 \times 10^{-25}$ Pa^{-3} yr^{-1}), and n = 3. The elastic properties of the material are E = 820 MPa and ν = 0.3636.

The model uses the same *UDEC* grid as that shown in Figure 1.7, and the same boundary conditions as those described in Section 1.6.3. The data file for this problem is given in Example 1.7. The analytical solution is included in this data file as the *FISH* function **powcyl**.

Example 1.7 Cylindrical cavity – WIPP model

```
model new
;file creep_07.dat
;
; WIPP Creep Model cylinder cavity test
;
model title ...
'WIPP Creep Model cylinder cavity test'
block config creep
block largestrain off

call 'cylinder.fis'
;
block tolerance corner-round-length .001
fish set @a_z = 10.0,
fish set @_a_radius = 1.0,
fish set @r_a = 20.0
@_geometry
@_rings
; use quarter symmetry
;
block delete range pos-x @_mod_size 0 pos-y @_mod_size @mod_size
block delete range pos-x @_mod_size @mod_size pos-y @_mod_size 0
```

```
block delete range pos-x 0 @ahalf pos-y 0 @ahalf
block contact join by-cont
@_gzones

block zone cmodel assign wipp
block zone property constant-gas 1.987 activation-energy 1.2e4 ...
  exponent 3 constant-d 5.52e-17
block zone property  constant-a 0 constant-b 0 ...
  creep-rate-critical 4.39e-8 temp 300
block zone property  bu = 1e9 sh .3e9 dens 2000
block edge apply stress -100e6,0,-100e6 ...
  range ann center 0 0 rad 19.9 20.1
block gridpoint apply vel-x 0 range pos-x -.1 .001
block gridpoint apply vel-y 0 range pos-y -.1 .001
block insitu stress -100e6,0,-100e6 szz -100e6
block gridpoint history vel-x  .5 0
block gridpoint history vel-y  0 .5
block creep timestep fix  0
block solve
model save 'creep_07a.sav'

;
hist reset
hist interval 100
block creep history time
block creep history timestep
block gridpoint history vel-x  .5 0
block gridpoint history vel-y  0 .5
;
; set creep time parameters
;
block creep timestep lower-bound 5e4
block creep timestep upper-bound 5e7
block creep timestep maximum 1.e-4
block creep timestep minimum 1e-7
block creep timestep auto
block creep timestep starting 1e-7
block cycle 1000000
model sav 'creep_07b.sav'

call 'powcyl.fis'
@compsol
model title 'Steady-state radial velocity vs distance from hole'
table 11 label  'analytical'
table 1  label  'UDEC'
model title 'Steady-state radial stress vs distance from hole'
```

```
table 2  label 'UDEC'
table 12 label 'analytical'
model title 'Steady-state hoop stress vs distance from hole'
table 13 label 'analytical'
table 3  label 'UDEC'
model title 'Steady-state out-of-plane vs distance from hole'
table 4  label 'UDEC'
table 14 label 'analytical'
model sav 'creep_07c.sav'

ret
```

The results of this example are identical to those for the power-law test in Section 1.6.3. Figure 1.14 compares the analytical solution for radial velocity to the *UDEC* results, and Figure 1.15 compares the analytical solution for radial, hoop and out-of-plane stresses to the *UDEC* results.

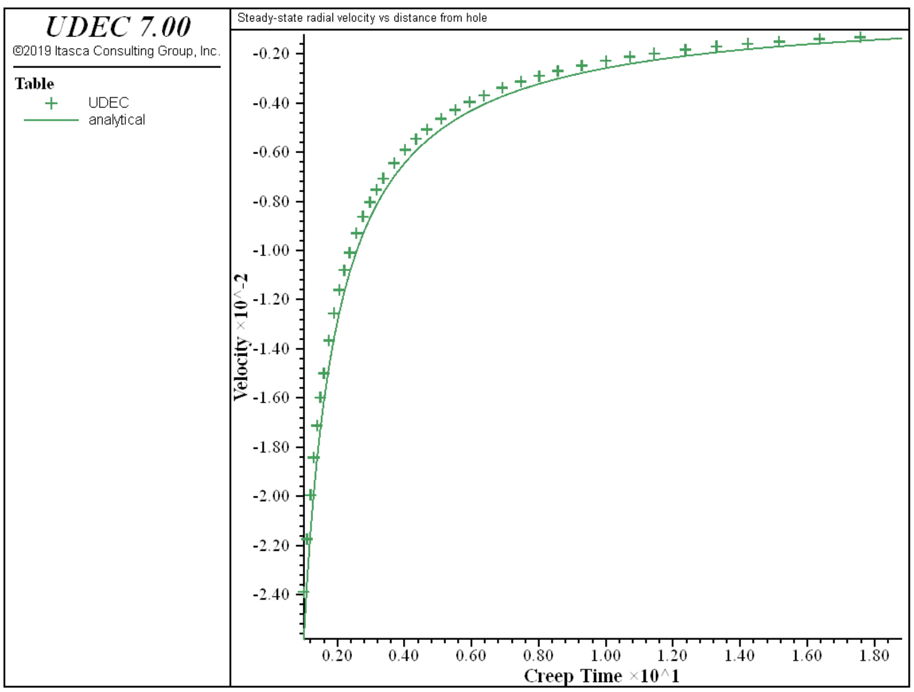

Figure 1.14 Comparison of radial velocity at steady state

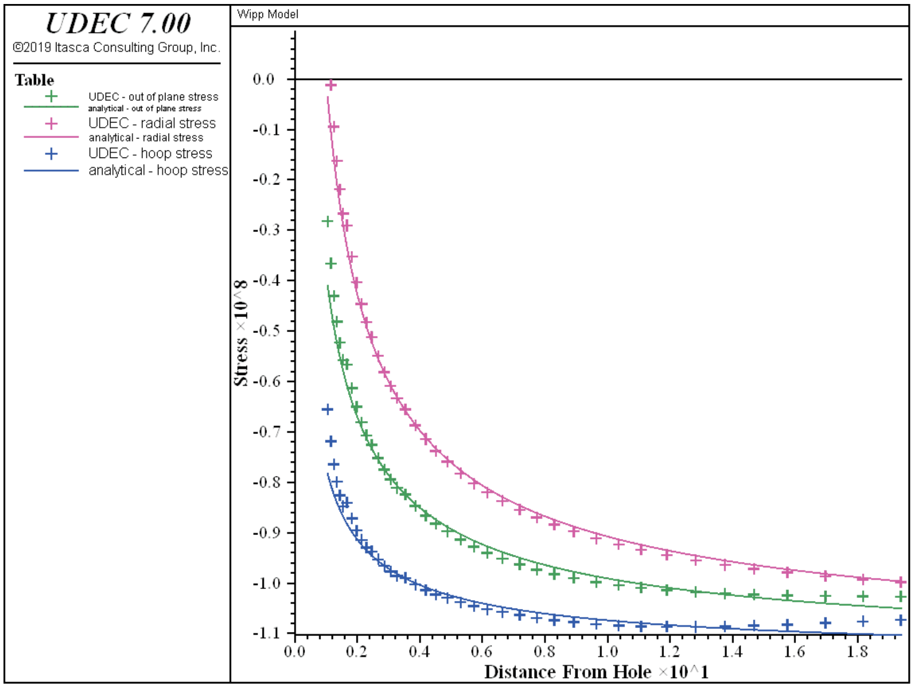

Figure 1.15 Comparison of radial, hoop and out-of-plane stress at steady state

1.6.5 Viscoelastic Response of the Burgers-Creep Viscoplastic Model

The Maxwell and Kelvin viscoelastic behavior of the viscoplastic model (**block zone cmodel assign burger-mohr**) are compared to the analytical solution for an oedometer test in the two following examples. In Example 1.8, the Kelvin cell logic is not taken into account by the model since no value is assigned to the property **viscosity-kelvin**. The cohesion property is set to a high value to prevent triggering of the plasticity logic. The initial timestep is set to a small value ($\Delta t = 10^{-3}$) compared to the ratio of viscosity over shear modulus ($\eta^M/G^M = 2.0$). With the choice of automatic creep timestep parameter settings used in the example, the timestep doubles when the out-of-balance force is less than 10^{-8}, until $\Delta t = 0.1$. A state of hydrostatic stress is reached at the end of the test.

Example 1.9 is the equivalent of Example 1.8, in which the Kelvin cell and elastic behavior of the Maxwell cell are active. Since no value is assigned to the property **viscosity-maxwell**, the model ignores the viscous component of the Maxwell cell. The cohesion property is again set to a high value. The timestep is set to a constant value (10^{-3}), a small value compared to the ratio η^M/G^M = 1.0. At the end of the simulation, the Kelvin and Maxwell shear moduli are acting in series (long-term behavior).

Example 1.8 Oedometer test on a Maxwell material – comparison with the analytic solution

```
model new
;File:creep_08.dat
model Title 'Odeometer test on a Maxwell material'
fish def ini_cons
  c_bu = 1.
  c_sh = 1.
  c_vi = 2.
  c_pr = -1.
  a1  = c_bu + 4. * c_sh / 3.
  c_a = 2. * c_sh / a1
  c_b = c_a * c_bu / (2. * c_vi)
  c_c = c_a * 2. / 3.
  c_d = -c_pr / c_bu
end
@ini_cons
;
block config creep
block largestrain off
block tolerance corner-round-length 1E-3
block tolerance minimum-edge-length 2E-3
block create polygon 0,0 0,1 1,1 1,0
block zone gen edge 1.0
block zone group 'mat1'
block zone cmodel assign burgers-mohr density 1 bu=@c_bu ...
  shear-maxwell=@c_sh cohesion 1E20 tension 1E20 ...
  viscosity-maxwell=@c_vi range group 'mat1'
bl edg app stress 0.0,0.0,@c_pr range pos-x 0,1 pos-y 0.99,1.01
bl grid apply vel-x 0
bl grid apply vel-y 0 ran p-x -6.786E-3,1.0068 p-y -9.405E-3,7.619E-3
; --- fish functions ---
fish def ana_eyy
  ana_eyy = -c_d  * (1. - c_c * math.exp(-c_b * bl.cr.time.total))
  ana_sxx =  c_pr * (1. - c_a * math.exp(-c_b * bl.cr.time.total))
  ana_syy =  c_pr
end
fish def _sxx
  _sxxt = 0
  _syy = 0
  _count = 0
  _zi = bl.zone(block.head)
  loop while _zi # 0
    _sxxt = _sxxt + bl.zo.str.xx(_zi)
    _syy  = _syy  + bl.zo.str.yy(_zi)
```

```
    _count = _count + 1
    _zi = bl.zone.next(_zi)
  endloop
  _sxx = _sxxt / _count
  _syy = _syy / _count
end

; --- elastic equilibrium ---
block cycle 1000
history interval 100
fish history @_sxx
fish history @ana_sxx
fish history @_syy
fish history @ana_syy
block gridpoint history disp-y 0.0,1.0
fish history @ana_eyy
block creep history time-total
; --- viscous behaviour ---
block creep timestep fix =1.e-3
block cycle 1000
block creep timestep maximum=0.1
block creep timestep minimum=0.0010
block creep timestep lower-bound=1.0E-8
block creep timestep upper-bound 1
block creep timestep upper-mult = 1.0
block creep timestep lower-mult = 2.0
block creep timestep start 1e-3
block creep timestep auto
block solve age 25.0
model save 'creep_08.sav'
ret
```

Example 1.9 Oedometer test on a Kelvin material – comparison with the analytic solution

```
model new
;File:creep_09.dat
model Title 'Odeometer test on a Kelvin material'
fish def ini_cons
  c_bu  = 2.
  c_sh  = 2.
  c_ksh = 1.
  c_kvi = 1.
  c_pr  = -1.
  a     = 3. * (c_bu + 4. * c_sh / 3.) / (2. * c_sh)
  al    = c_ksh + 3. * c_bu / (2. * a)
```

```
  c_a = al / c_kvi
  c_b = 3. * c_pr / (a * a * al)
  c_c =  c_pr / (c_bu + 4. * c_sh / 3.)
  c_d = 3. * c_bu
end
@ini_cons
;
block config creep
block largestrain off
block tolerance corner-round-length 1E-3
block tolerance minimum-edge-length 2E-3
block create polygon 0,0 0,1 1,1 1,0
block zone gen edge 1.0
block zone group 'mat1'
block zone cmodel assign burger-mohr density 1 bu=@c_bu ...
  shear-maxwell=@c_sh shear-kelvin=@c_ksh viscosity-kelvin=@c_kvi ...
  cohesion 1E20 tension 1E20 range group 'mat1'
bl edg app stress 0.0,0.0,@c_pr range pos-x 0,1 pos-y 0.99,1.01
block gridpoint apply velocity-x 0
block gridpoint apply velocity-y 0 ...
  range pos-x -6.786E-3,1.0068 pos-y -9.405E-3,7.619E-3
; --- fish functions ---
fish def ana_eyy
  val     =  c_b  * (1. -  math.exp(-c_a * bl.cr.ti.total)) + c_c
  ana_eyy =  val
  ana_sxx = 0.5 * (c_d * val - c_pr)
  ana_syy =  c_pr
end
fish def _sxx
  _sxxt = 0
  _syy = 0
  _count = 0
  _zi = bl.zone(block.head)
  loop while _zi # 0
    _sxxt = _sxxt + bl.zo.str.xx(_zi)
    _syy = _syy + bl.zo.str.yy(_zi)
    _count = _count + 1
    _zi = bl.zone.next(_zi)
  endloop
  _sxx = _sxxt / _count
  _syy = _syy / _count
end
; --- elastic equilibrium ---
block cycle 1000
hist interval = 50
fish history @_sxx
```

```
fish history @ana_sxx
fish history @_syy
fish history @ana_syy
block gridpoint history disp-y 0.0,1.0
fish history @ana_eyy
block creep history crtime
; --- viscous behaviour ---
block creep timestep fix =1.e-3
block cycle 3000

model save 'creep_09.sav'

ret
```

Figures 1.16 to 1.19 show the agreement between analytical solutions and numerical predictions for stresses and strains.

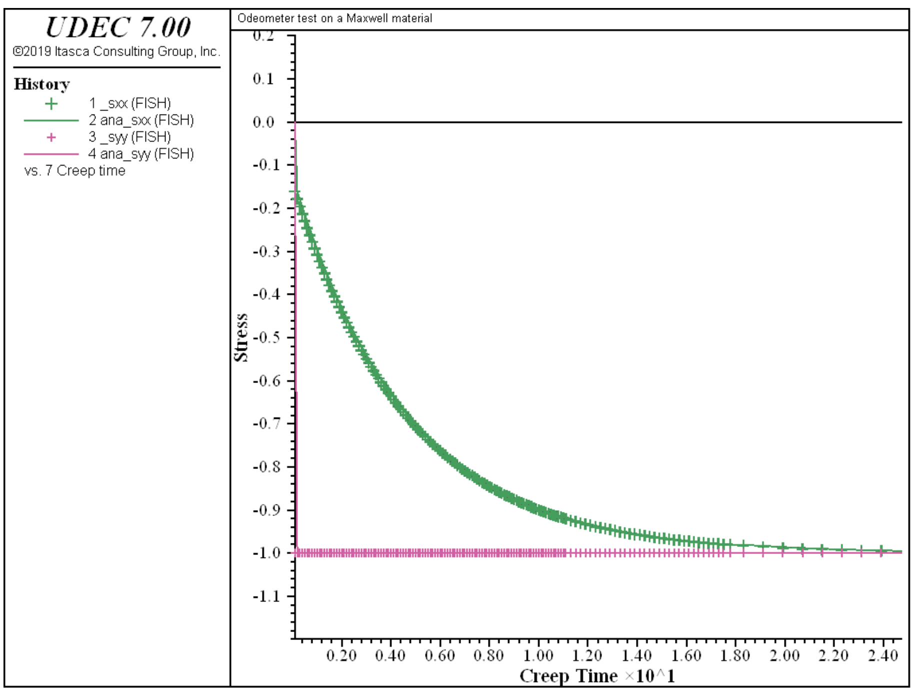

Figure 1.16 Comparison between analytical and numerical stress values – Maxwell cell

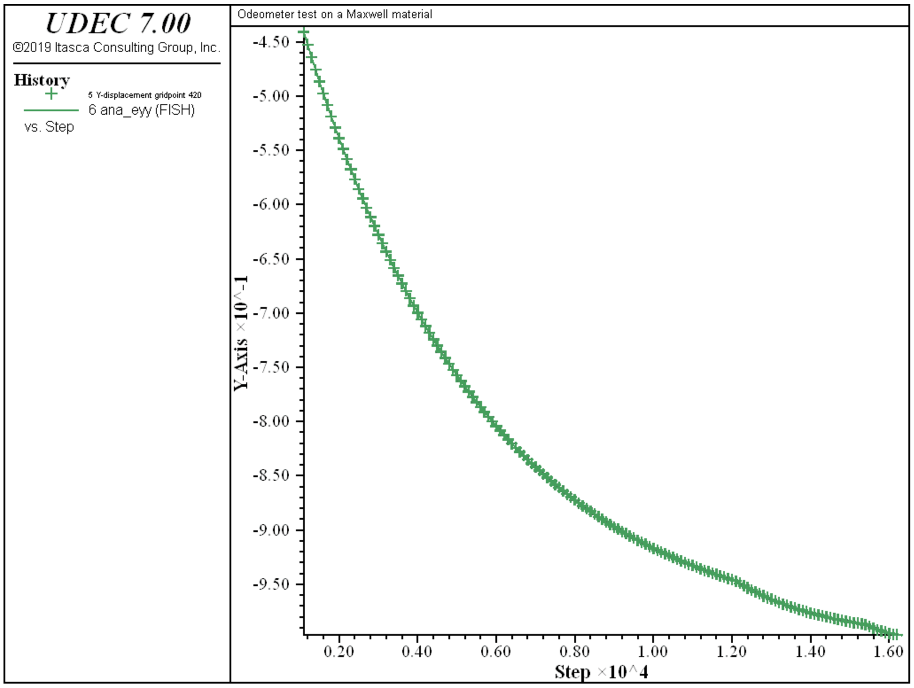

Figure 1.17 Comparison between analytical and numerical strain values – Maxwell cell

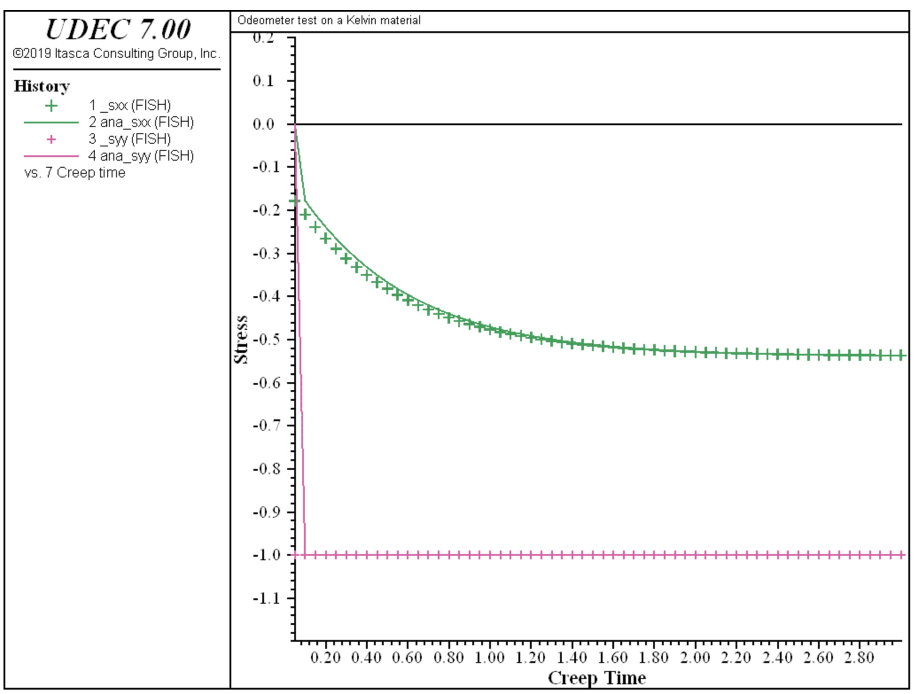

Figure 1.18 Comparison between analytical and numerical stress values – Kelvin cell

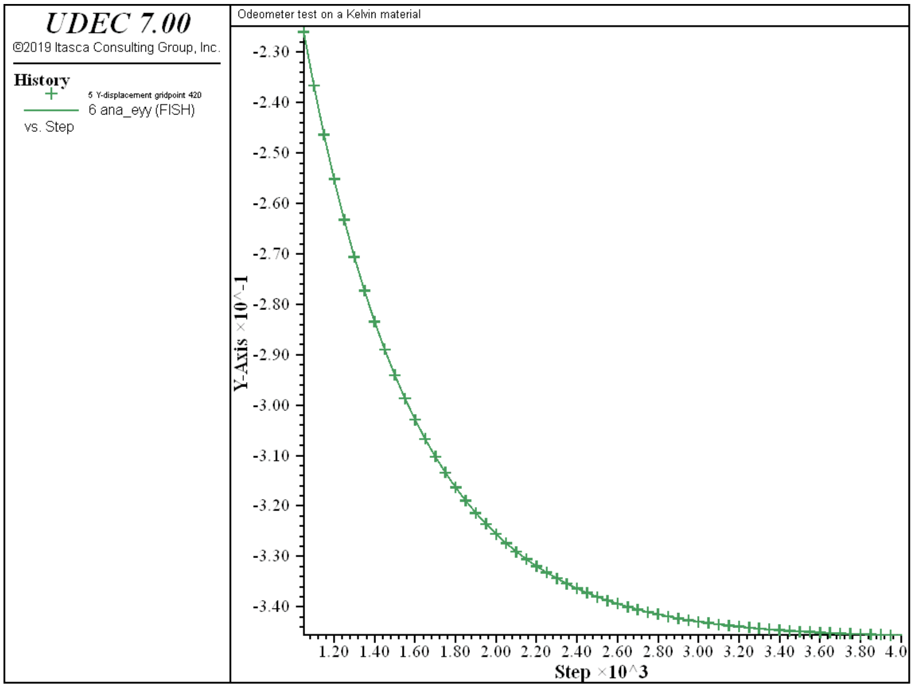

Figure 1.19 Comparison between analytical and numerical strain values – Kelvin cell

1.6.6 Viscoplastic Response of the Burgers-Creep Viscoplastic Model

The responses of the models **burger-mohr** and **mohr** are compared in an unconfined compression test. The viscous component of the Maxwell cell in **burger-mohr** is not activated in Example 1.10.

In the first part of the test, a horizontal compressive velocity of magnitude 10^{-7} (measured in units of distance per step) is applied on both sides of the model for a total of 1500 steps. The timestep is set to 10^{-3}, a small value compared to the ratio η^K/G^K of 10.

The Mohr-Coulomb model predicts that shear yielding will take place when the horizontal stress reaches the value of $-2C\sqrt{N_\phi}$ ($\simeq -1.281 \times 10^6$). In the framework of this model, and up to incipient failure, we have

$$\sigma_{xx} = \alpha_1[1 - {\alpha_2}^2/{\alpha_1}^2]\epsilon_{xx}$$

with $\alpha_1 = K + 4/3G$, $\alpha_2 = K - 2/3G$ and, from the boundary conditions,

$$\epsilon_{xx} = -2\, v\, n/L$$

where v is the applied velocity magnitude, n is the number of steps elapsed to incipient failure, and L is the horizontal length of the sample.

The numerical results are presented in Figure 1.20. Note that the **burger-mohr** sample fails at the same stress level, but later in time, thus reflecting the effect of creep (at incipient failure, the time scale is about 0.75, a value that is not small compared to the characteristic time $\eta^K/G^K = 1.0$ of the creep process). When the loading rate is increased and the simulation is repeated for the same applied velocity and same number of steps but smaller creep timesteps, the responses of the two models become more similar. For a timestep of 10^{-5}, the effect of creep cannot be detected on the plot – see Figure 1.21 (at incipient failure, the creep time scale is now about 0.75×10^{-2}).

In the second part of the test, the compressive velocity is set to zero and the models are cycled for 1500 steps. While the **mohr-coulomb** sample stays at yield, the **burger-mohr** sample unloads as creep develops (see Figure 1.22). The interaction between creep and plastic flow may be appreciated by comparing the viscoplastic behavior in Figures 1.22 and 1.23: in the latest plot, more plastic flow (measured by **strain-shear-plastic**) is allowed to take place before the compressive velocity is set to zero and, subsequently, the magnitude of maximum creep unloading is reduced.

In the third part of the test, the samples are "reloaded" by application of a *UDEC* velocity of 10^{-5} for a total of 500 steps. At the end of the test, both samples are yielding at the same stress level (see Figure 1.24).

Note that no dimension is specified for the values quoted above; they may be interpreted in any consistent system of units, but are probably not representative, and are given only for illustration purposes.

Example 1.10 Comparison of the Burgers-creep viscoplastic model and the Mohr-Coulomb model

```
model new
;File:creep_10.dat
mode Title ...
'Compression test on Burger viscoplastic and Mohr material'
;
block config creep
block largestrain off
block tolerance corner-round-length 9E-3
block tolerance minimum-edge-length 1.8E-2
block create polygon 0,0 0,1 9,1 9,0
block cut crack (3,0) (3,1)
block cut crack (6,0) (6,1)
block zone gen quad 4.0
block zone group 'mo'

block zone group 'cv' range atblock (1.5,0.5)
block zone cmodel assign mohr-c density 2.5E3 bulk 1.19E10 ...
  shear 1.1E10 friction 44 cohesion 2.72E5 tension 2E5 range group 'mo'
```

```
block zone cmodel assign burger-mohr density 2.5E3 bulk 1.19E10 ...
  shear-kelvin 1.1E10 shear-maxwell 1.1E10 viscosity-kelvin 1.1E10 ...
  cohesion 2.72E5 friction 44 tension 2E5 range group 'cv'
block contact prop mat 1 st-n 1e1 st-s 1e1
block zone group 'Null' range atblock (4.5,0.5)
block change model 0 range group 'Null'
block prop mat 1 bu 1e10 sh .7e10 dens 2500
;block zone cmodel assign null range group 'Null'
;block zone prop dens 2.5e3 bu 1.19e10 sh 1.1e10 range group 'Null'
; --- fish function ---
fish def squez
  svp = squez_vel
  svm = -svp
  command
    bl grid apply velocity-x @svp range pos-x -0.1,0.1 pos-y -0.1,1.1
    bl grid apply velocity-x @svm range pos-x  2.9,3.1 pos-y -0.1,1.1
    bl grid apply velocity-x @svp range pos-x  5.9,6.1 pos-y -0.1,1.1
    bl grid apply velocity-x @svm range pos-x  8.9,9.1 pos-y -0.1,1.1
  end_command
end
fish set @squez_vel=1.27e-3
@squez
fish def _sxx
  _sxxt = 0
  _count = 0
  _bi = bl.near(_x, _y)
  _zi = bl.zone(_bi)
  loop while _zi # 0
    _sxxt = _sxxt + bl.zo.str.xx(_zi)
    _count = _count + 1
    _zi = bl.zone.next(_zi)
  endloop
  _sxx = _sxxt / _count
end
fish def _sxx1
  _x = 1.5
  _y = 0.5
  _sxx
  _sxx1 = _sxx
  _x = 7.5
  _y = 0.5
  _sxx
  _sxx2 = _sxx
end
block gridpoint history disp-x  0.0,0.0
fish history @_sxx1
```

```
fish history @_sxx2
block creep history time-total
model save 'creep_10a.sav'


model restore 'creep_10a.sav'
block creep timestep fix =0.0010
block cycle  1500
model save 'creep_10b.sav'



model restore 'creep_10a.sav'
block creep timestep fix  = 1e-5
block cycle 1500
model save 'creep_10c.sav'


model restore 'creep_10a.sav'
block creep timestep fix  = 1e-3
block cycle 1500
fish set @squez_vel = 0
@squez
block cycle 1500
model save 'creep_10d.sav'


model restore 'creep_10a.sav'
block creep timestep fix  = 1e-3
block cycle 3000
fish set @squez_vel = 0.
@squez
block cycle 1500
model save 'creep_10e.sav'


model restore 'creep_10a.sav'
block creep timestep fix  = 1e-3
block cycle 1500
fish set @squez_vel = 0.
@squez
block cycle 1500
;
fish set @squez_vel = 1.e-5
@squez
block cycle 2000
```

```
;
fish set @squez_vel = 0.
@squez
block cycle 1500
model save 'creep_10f.sav'
```

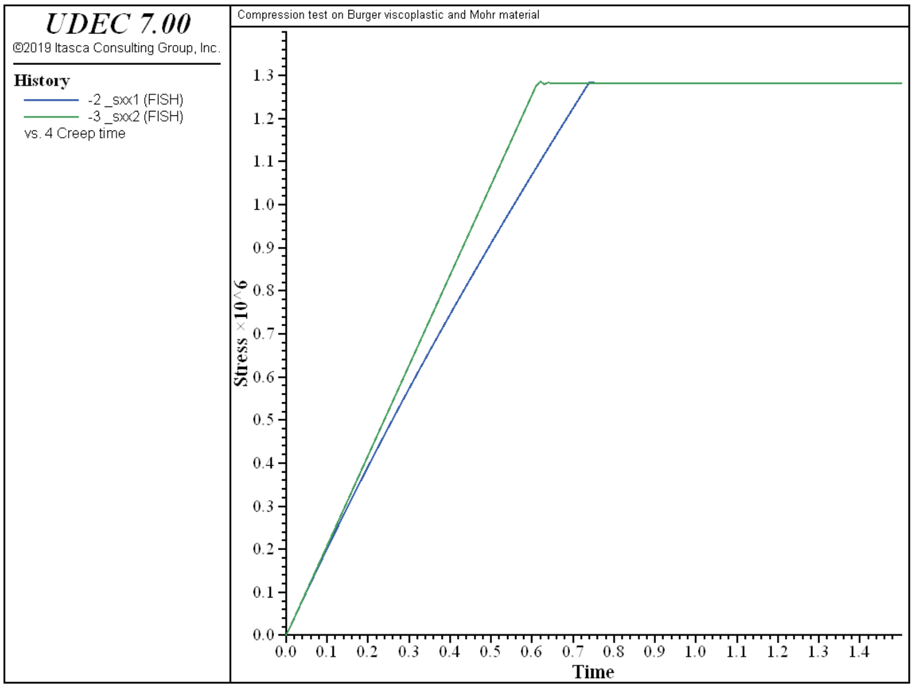

Figure 1.20 Experiment 1: Horizontal stress versus time for slow compression test

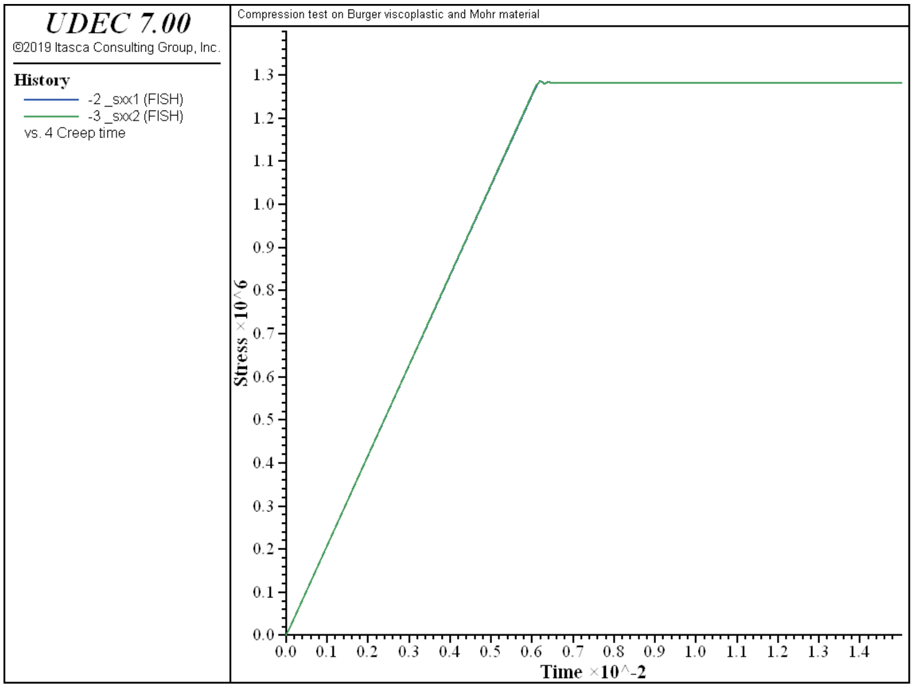

Figure 1.21 Experiment 2: Horizontal stress versus time for rapid compression test

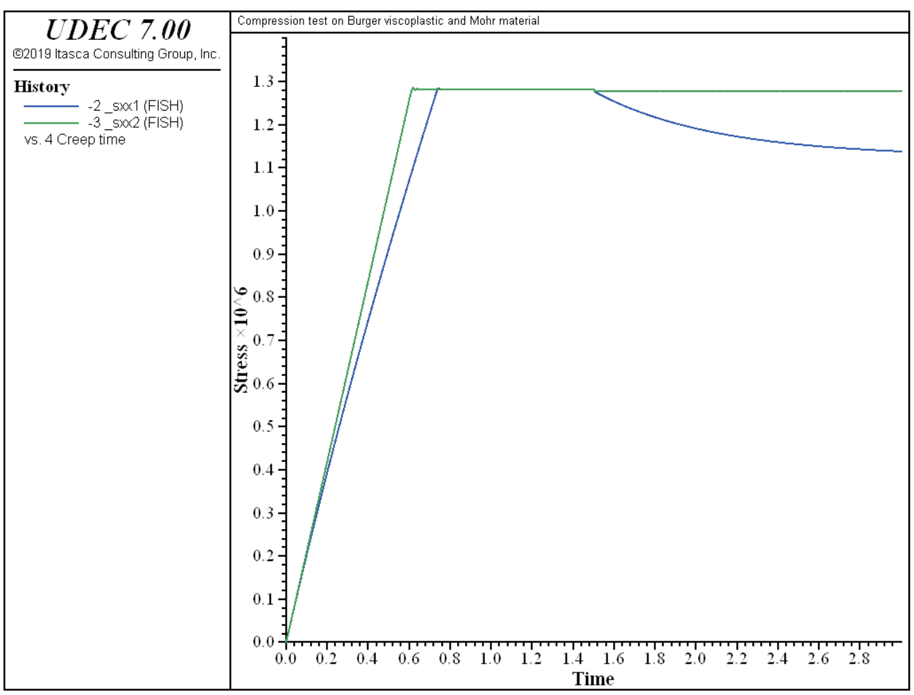

Figure 1.22 Experiment 3: Creep unloading after less plastic flow

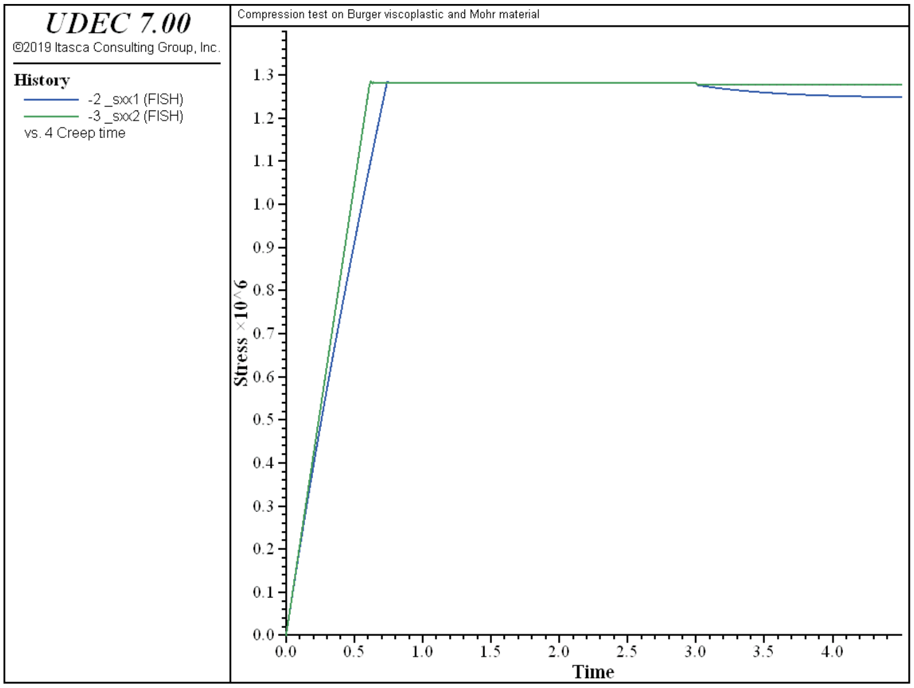

Figure 1.23 Experiment 4: Creep unloading after more plastic flow

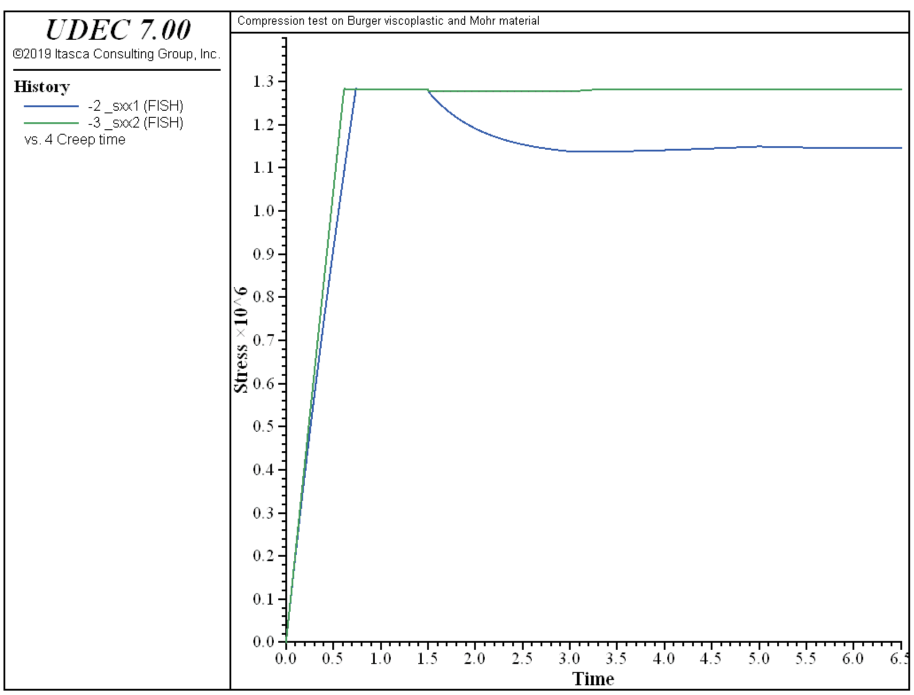

Figure 1.24 Experiment 5: Horizontal stress versus time for several loading excursions

1.6.7 Plastic Response of the WIPP-Creep Viscoplastic Model

The viscoplastic model (**block zone cmodel assign wipp-drucker**) is compared to the Drucker-Prager model (**block zone cmodel assign drucker**). If creep is inhibited in the viscoplastic model, the same behavior is exhibited in both models. Example 1.11 exercises models **wipp-drucker** and **drucker-prager** by applying a complex strain history to both, and comparing corresponding histories of all stress components. The maximum error (normalized to the maximum recorded stress component) is printed at the end. It is found that the maximum normalized error is 0.007%, which is within the bounds of machine accuracy. Note that creep is active throughout the simulation, but the effects of creep are small because the timestep is set to 0.1 second. If the simulation is continued, the mean stress becomes tensile. In this case, the apparent error increases to 0.1%, but this is simply due to the fact that the two models are alternating between shear and tensile yield at different points in time.

If Example 1.11 is repeated with timesteps of 10 and 20 seconds, the effects of creep will be increasingly more significant in comparison with plasticity effects. There is a general decay of stress components, relative to the Drucker-Prager model stress components, when creep is active.

Example 1.11 Comparison of the viscoplastic model and the Drucker-Prager model

```
model new
;File:creep_11.dat
block config creep
block largestrain off
block tolerance corner-round-length 2E-3
block tolerance minimum-edge-length 4E-3
block create polygon 0,0 0,1 2,1 2,0
block zone gen edge 1.0
block zone group 'pw' range pos-x 0,1 pos-y 0,1
block zone cmodel assign wipp-drucker density 2.6E3 bulk 2.07E10 ...
  shear 1.24E10 activation-energy 1.2E4 constant-a 4.56 ...
  constant-b 127 constant-d 5.79E-36 constant-gas 1.987 ...
  exponent 4.9 temperature 300 friction-drucker 0.5 ...
  cohesion-drucker 1E4 dilation-drucker 0.3 tension 1e15 ...
  creep-rate-critical 5.39E-8 range group 'pw'
block zone group 'dr' range pos-x 1,2 pos-y 0,1
block zone cmodel assign drucker-prager density 2.6E3 bulk 2.07E10 ...
 shear 1.24E10 friction-drucker 0.5 ...
 cohesion-drucker 1E4 dilation-drucker 0.3 tension 1E15 range group 'dr'
block insitu stress 100.0,0.0,100.0 szz 100.0
block gridpoint apply vel-x 0
block gridpoint apply vel-y 0
fish def _stress
  _z1 = bl.zone.near(_x+0.25,0.5)
  _z2 = bl.zone.near(_x+0.5,0.75)
  _z3 = bl.zone.near(_x+0.75,0.5)
```

```
  _z4 = bl.zone.near(_x+0.5,0.25)
;
  _sxx = (bl.zo.str.xx(_z1)+bl.zo.str.xx(_z2)+bl.zo.str.xx(_z3) ...
         +bl.zo.str.xx(_z4))/4.0
  _syy = (bl.zo.str.yy(_z1)+bl.zo.str.yy(_z2)+bl.zo.str.yy(_z3) ...
         +bl.zo.str.yy(_z4))/4.0
  _szz = (bl.zo.str.zz(_z1)+bl.zo.str.zz(_z2)+bl.zo.str.zz(_z3) ...
         +bl.zo.str.zz(_z4))/4.0
  _sxy = (bl.zo.str.xy(_z1)+bl.zo.str.xy(_z2)+bl.zo.str.xy(_z3) ...
         +bl.zo.str.xy(_z4))/4.0
end
fish def sig_0_wipp
  _x = 0
  _stress
  _sxx1 = _sxx
  _syy1 = _syy
  _szz1 = _szz
  _sxy1 = _sxy
  sig_0_wipp = (_sxx1 + _syy1 + _szz1)/3.0
  _x = 1
  _stress
  _sxx2 = _sxx
  _syy2 = _syy
  _szz2 = _szz
  _sxy2 = _sxy
  sig_0_dp   =(_sxx2 + _syy2 + _szz2)/3.0
end
fish def w_error ; determine normalized error between d-WIPP and D-P
  s_level   = math.max(s_level,math.abs(_sxx2))
  s_level   = math.max(s_level,math.abs(_syy2))
  s_level   = math.max(s_level,math.abs(_szz2))
  s_level   = math.max(s_level,math.abs(_sxy2))
  er1       = math.abs(_sxx1 - _sxx2)
  er2       = math.abs(_syy1 - _syy2)
  er3       = math.abs(_szz1 - _szz2)
  er4       = math.abs(_sxy1 - _sxy2)
  w_temp    = math.max(er1,er2)
  w_temp    = math.max(w_temp,er3)
  n_error   = math.max(w_temp,er4) / s_level ; normalize
  w_error   = n_error
  max_error = math.max(max_error,n_error)
end
fish set @s_level=0.0
fish set @max_error=0.0
block creep timestep fix =0.1 ; (small dt for no creep)
block creep history time
```

```
fish history @sig_0_wipp
fish history @sig_0_dp
fish history @w_error
fish history @s_level
fish history @_sxx1
fish history @_syy1
fish history @_szz1
fish history @_sxy1
fish history @_sxx2
fish history @_syy2
fish history @_szz2
fish history @_sxy2
fish def qqq
  oo = io.out(' Please wait ...')
end
;---> Execute a weird strain history ...
bl grid apply velocity-y -1.5E-4 range pos-x -0.1,2.1 pos-y 0.9,1.1
bl grid apply velocity-x  1.5e-4 range pos-x -0.1,2.1 pos-y 0.9,1.1
@qqq
block cycle 500
bl grid apply velocity-x  1.5e-4 range pos-x -0.1,2.1 pos-y 0.9,1.1
block cycle 400
bl grid apply velocity-x -1.5e-4 range pos-x -0.1,2.1 pos-y 0.9,1.1
block cycle 400
bl grid apply velocity-x  0.0
bl grid apply velocity-y  1.5E-4 range pos-x -0.1,2.1 pos-y 0.9,1.1
block cycle 200
bl grid app grad-vel 0.0,0.0 1.5e-5,0.0 7.5e-5,0
block cycle 400
bl grid app vel-grad 0.0,0.0 -1.5e-5,0.0 7.5e-5,0.0
block cycle 400
bl grid app vel-grad 0.0,0.0 7.5e-5,0.0 0.0,7.5e-5
block cycle 1500
fish def _show ; show the max error
  oo = io.out('Max normalized error = '+string(max_error*100.0)+' percent')
end
@_show
model save 'creep_11.sav'

ret
```

1.6.8 Compression Test with the WIPP-Creep Viscoplastic Model

A compression test is performed with the viscoplastic model (**wipp-drucker**) to demonstrate that the model is capable of simulating localization, given an appropriate loading rate. Example 1.12 causes *UDEC* to perform an unconfined compression test on a sample in which one of the material strength-parameters (**cohesion-drucker**) reduces with increasing plastic strain. Under a low strain rate, the response is monotonic, and the sample deforms in a uniform manner. Figure 1.25 shows contours of maximum shear strain for a test performed over a period of 500,000 seconds. If a similar test is performed at a rate ten times faster (i.e., over 50,000 seconds), localization occurs, even though the same boundary displacement is applied. Figure 1.26 shows that shear bands have formed in the rapid-loading case. Although the maximum load is nearly the same for the two loading-rate tests, the latter test exhibits global softening behavior.

Example 1.12 Compression test using the WIPP-creep viscoplastic model

```
model new
;File:creep_12.dat
fish def soften ; strain-softening law
  while_stepping
  soft_rep = soft_rep + 1
  if soft_rep >= 10
    soft_rep = 0
    _zi = bl.zone(block.head)
    loop while _zi # 0
      eplas = bl.zone.prop(_zi, 'e_plastic')
      if eplas # 0.0
        rat_fac = 1.0 - eplas / 2e-4
        if rat_fac > 0.0
          bl.zone.prop(_zi,'cohesion-drucker') = max_strength * rat_fac
        else
          bl.zone.prop(_zi,'cohesion-drucker') = 0.0
        endif
      endif
      _zi = bl.zone.next(_zi)
    endLoop
  endif
end
fish def _load ; model save load & disp. for histories
  _sum = 0.0
  loop i (0,20)
    _sum = _sum - bl.gp.force.y(bl.gp.near(i,0.1))
    _sum = _sum + bl.gp.force.y(bl.gp.near(i,59.9))
  endLoop
  _load = _sum / 2.0
  _displacement = bl.gp.disp.y(bl.gp.near(0,0)) ...
     - bl.gp.disp.y(bl.gp.near(0,60))
```

```
end
fish set @max_strength = 3.5e6,
fish set @soft_rep=0
block config creep
block largestrain off
block tolerance corner-round-length 6E-2
block tolerance minimum-edge-length 0.12
block create polygon 0,0 0,60 20,60 20,0
block zone gen quad 1
block zone group 'pw'
block zone cmodel assign wipp-drucker density 2.6E3 bulk 2.07E10 ...
  shear 1.24E10 activation-energy 1.2E4 constant-a 4.56 ...
  constant-b 127 constant-d 5.79E-36 creep-rate-critical 5.39E-8 ...
  constant-gas 1.987 exponent 4.9 temperature 300 ...
  friction-drucker 0.75 cohesion-drucker @max_strength ...
  dilation-drucker 0 tension 1E15 range group 'pw'
block grid apply vel-x 0 range p-x -1.2357,21.1571 p-y 58.9143,60.8
block grid apply vel-x 0 range p-x -1.0786,20.6857 p-y -0.8786,0.6143
block creep history time
fish history @_load
fish history @_displacement
model save 'creep_12.sav'


; fast option
model restore 'creep_12.sav'
bl grid apply vel-y -1.17 range p-x -0.7643,20.45   p-y 59.4643,60.7214
bl grid apply vel-y  1.17 range p-x -0.8429,20.6071 p-y -0.8,0.6929
block creep timestep fix =10.0
block gridpoint initial vel-y 1.17E-1 grad 0.0,-3.91E-3
block cycle  5000
model save 'fast.sav'


; slow option
model restore 'creep_12.sav'
bl grid app vel-y -1.17e-1 ran p-x -0.7643,20.45   p-y 59.4643,60.7214
bl grid app vel-y  1.17e-1 ran p-x -0.8429,20.6071 p-y -0.8,0.6929
block creep timestep fix =100.0
block gridpoint initial vel-y 1.17E-2 grad 0.0,-3.9E-4
block cycle  50000
model save 'slow.sav'

ret
```

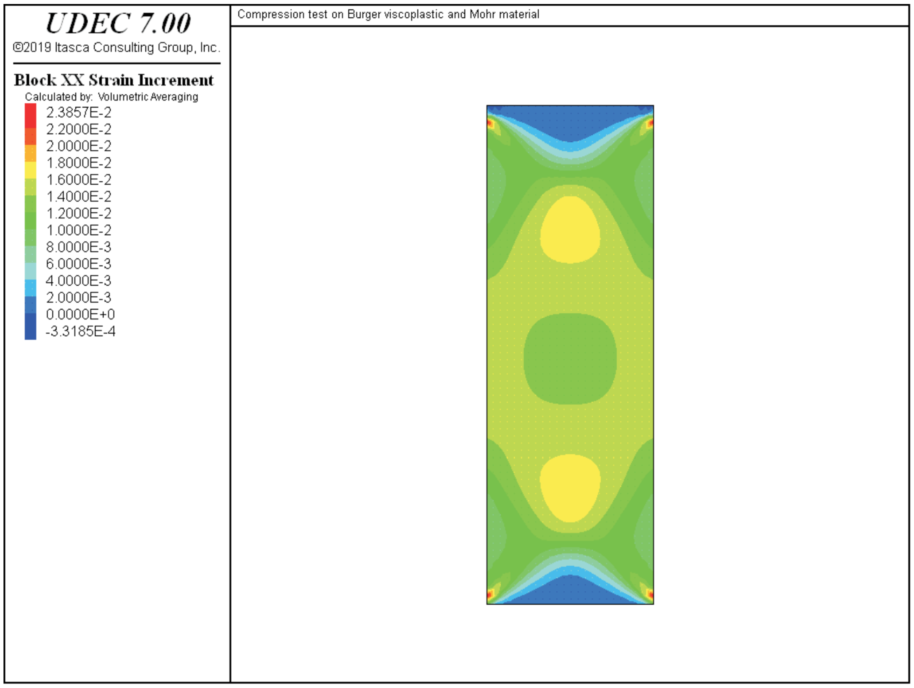

Figure 1.25 Contours of maximum shear strain increment for slow compression test

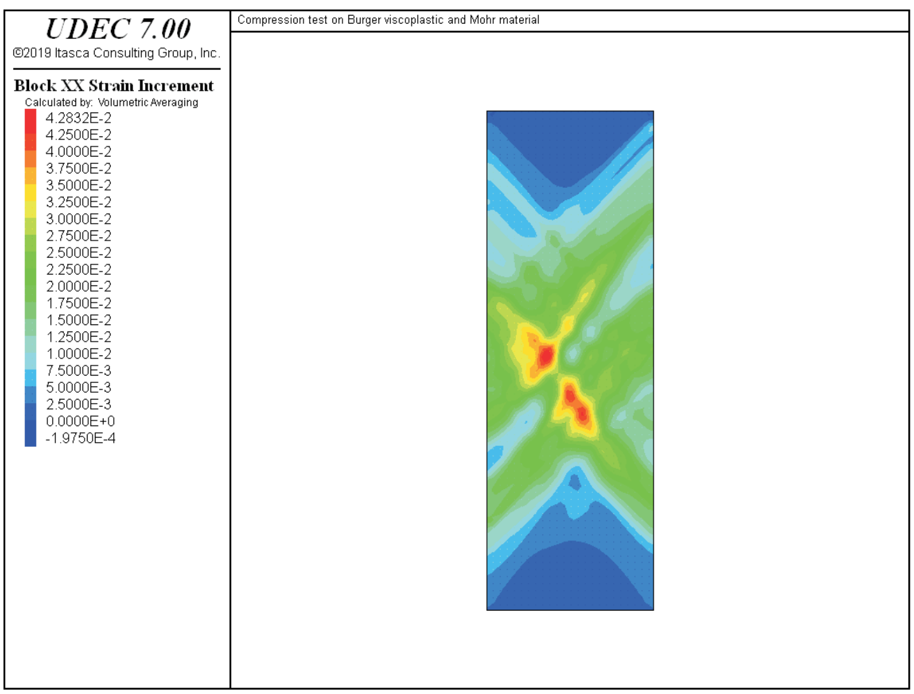

Figure 1.26 Contours of maximum shear strain increment for rapid compression test

1.6.9 Creep Response of a Bedded Salt Formation

A series of rooms is excavated in bedded salt at a depth of 646 m. A layer of anhydrite is located below the excavations, and clay seams are located above and below the excavations. The stratigraphy is shown in Figure 1.27, and is based upon stratigraphic information from Morgan et al. (1981). The stratigraphy and excavations reflect those of the Waste Isolation Pilot Plant near Carlsbad, New Mexico.

The objective of the *UDEC* analysis is to investigate the time-dependent response of the bedded salt as the rooms are excavated (i.e., room closure). This response will be affected by the presence of the clay layers and the anhydrite layer below the excavation floor.

Different room sizes can be investigated with the *UDEC* model. In this example, the rooms are 10 m (33 ft) wide and 4 m (13 ft) high, with room centers 40 m (131 ft) apart. This corresponds to an extraction ratio of 0.25.

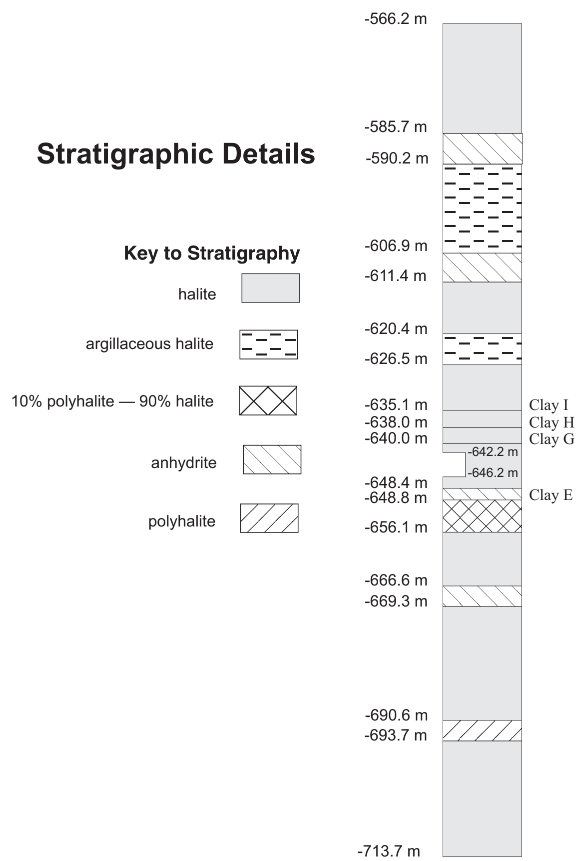

Figure 1.27 Model stratigraphy

Three bedded salt deposits are defined: halite, argillaceous-halite and a composite of 10% polyhalite and 90% halite. Both viscoelastic and viscoplastic behaviors are assumed to characterize the time-dependent response of the different salt layers. Non-salt anhydrite and polyhalite layers are also present, and are assumed to behave as Mohr-Coulomb materials. The clay layers are considered weakness planes with a Coulomb shear strength limit. Tables 1.2 and 1.3 summarize the material parameters.

The in-situ stress is lithostatic, with the vertical stress equal to the weight of the overburden material.

Table 1.2 Elastic, strength and creep properties of salt materials

	HALITE	ARGILLACEOUS-HALITE	10% POLYHALITE-90% HALITE
bulk modulus, K	20.7 GPa	20.7 GPa	22.1 GPa
shear modulus, G	12.4 GPa	12.4 GPa	13.2 GPa
WIPP constant, A	4.56	4.56	4.56
WIPP constant, B	127.0	127.0	127.0
WIPP constant, D	5.79 10^{-36} Pa^{-n} s^{-1}	1.74 $^{-35}$ Pa^{-n} s^{-1}	5.21 10^{-36} Pa^{-n} s^{-1}
WIPP exponent, n	4.90	4.90	4.90
activation energy, Q	12,000 cal/mol	12,000 cal/mol	12,000 cal/mol
crit. s-s creep rate, $\dot{\epsilon}^*_{ss}$	5.39 10^{-8} s^{-1}	5.39 10^{-8} s^{-1}	5.39 10^{-8} s^{-1}
D-P parameter, k_ϕ	5.0 MPa		
D-P parameter, q_ϕ	0.5		
D-P parameter, q_ψ	0.0		

Table 1.3 Elastic and strength properties of non-salt materials

	ANHYDRITE	CLAY SEAMS
bulk modulus, K	83.4 GPa	
shear modulus, G	27.8 GPa	
interface normal stiffness, k_n		1000 GPa/m
interface shear stiffness, k_s		50 GPa/m
friction coefficient, ϕ	29°	5°
cohesion, c	27 MPa	0.0

Figure 1.28 shows the *UDEC* model for this example. The regular geometry of the excavations and stratigraphy allows planes of symmetry to be specified vertically through the pillar center (i.e., pillar between adjacent rooms) and the center of a room. The geometry also allows plane-strain conditions to be considered.

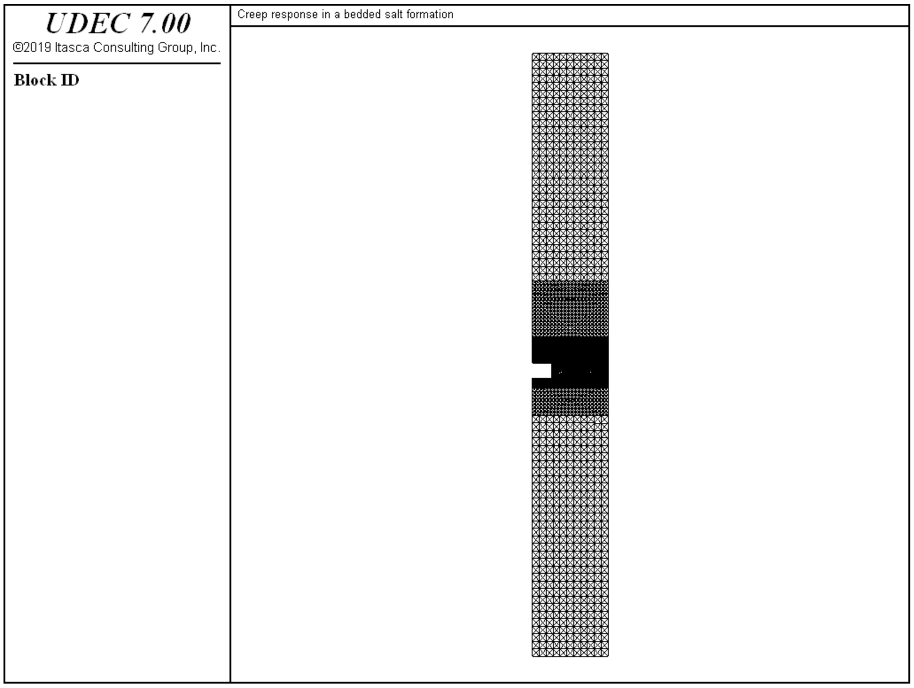

Figure 1.28 UDEC grid for bedded salt formation

Planes of symmetry are always planes of zero shear stress. Therefore, the symmetry planes are given roller boundaries. It is important that the upper and lower horizontal boundaries are far enough removed from the excavation to minimize boundary effects of the predicted results. The lower horizontal boundary, while not a plane of symmetry, is assigned roller boundaries. If far enough removed, the effects of fixed or rollered conditions for this boundary will be similar. The upper horizontal boundary is specified as a stress boundary, with a vertical stress equivalent to the weight of the material above this boundary.

The WIPP-reference creep model (**block zone cmodel assign wipp**) is assigned to all salt materials. Four interfaces are created in the model to represent the bedding planes, identified as clay E, clay G, clay H and clay I, in Figure 1.27. The material models and interfaces are shown in Figure 1.29.

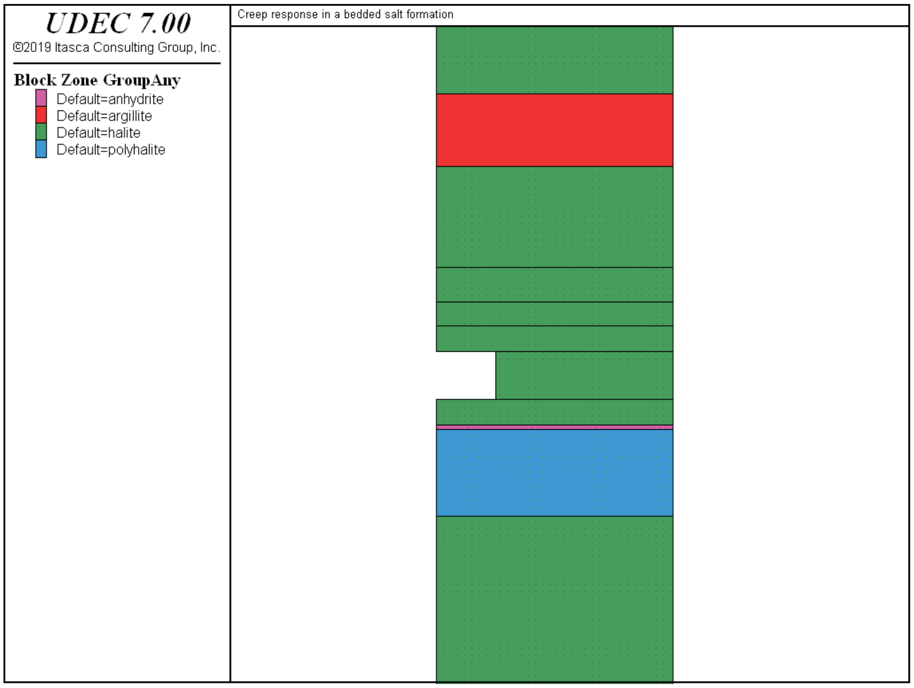

Figure 1.29 Material models

The analysis is done in three stages. In Stage (1) (i.e., prior to room excavation) the model is run to initial equilibrium. In Stage (2), the excavation takes place. This is done instantly, and the model is subsequently run to equilibrium without creep. Since in reality the excavation process is relatively fast, this is a reasonable approach. Only a minimal amount of creep can occur during the short excavation period. In Stage (3), the effect of creep is invoked, and the model is run for a period of one year. The vertical displacements in the roof and floor, and the horizontal displacement at the springline, are monitored during the one-year period.

The room closure is indicated by the convergence history plots in Figure 1.30.

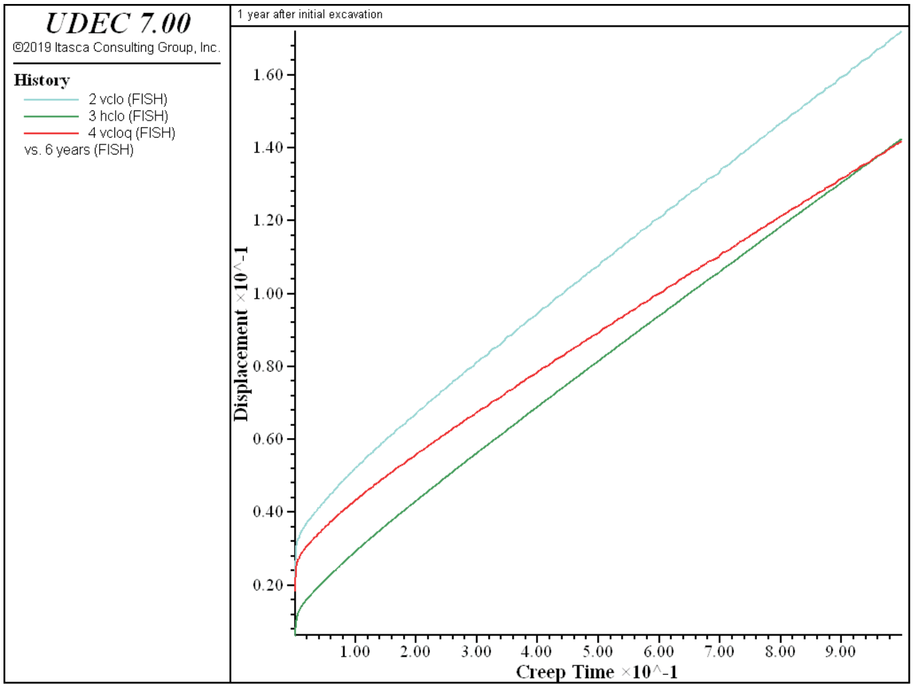

Figure 1.30 ***Room closure histories: vertical convergence at two locations (`vclo` and `vcloq`); and horizontal convergence (`hclo`) at the room mid-height (x-axis units are years, and y-axis units are inches)***

The *UDEC* data file is listed in Example 1.13. The *UDEC* model contains 10,296 zones, and requires approximately 1.6 MB RAM.

Example 1.13 Creep response of a bedded salt formation

```
model new
;file creep_13.dat
model title 'Creep response in a bedded salt formation'
block config creep
block tolerance corner-round-length 0.01
block contact tolerance overlap .1
block create polygon 0,-80 0 80 20 80 20 -80
; room
block cut crack 0.0,-2.2  20.0,-2.2 join
block cut crack 0.0,-6.2  20.0,-6.2 join
block cut crack 5.0,-6.2  5.0,-2.2  join
;
; geology
;
block cut crack 0.0,19.6  20.0,19.6    ; top of argillaceous halite
```

```
block cut crack 0.0,13.5  20.0,13.5    ; bottom of argillaceous halite
block cut crack 0.0,4.9   20.0,4.9     ; clay I
block cut crack 0.0,2.0   20.0,2.0     ; clay H
block cut crack 0.0,0.0   20.0,0.0     ; clay G
block cut crack 0.0,-8.8  20.0,-8.8    ; clay E
block cut crack 0.0,-8.4  20.0,-8.4    ; top of anhydrite
block cut crack 0.0,-16.1 20.0,-16.1   ; bottom of polyhalite
;
; zoning
;
hide range pos-y -16.1 19.6
block zone gen quad 2
block seek
hide range pos-y -8.8 4.9
block zone gen quad 1
block seek
block hide
block seek range pos-y -8.8 -8.4
block zone gen quad .3 .1
block seek
block zone gen quad .5

block zone group 'halite'
block zone group 'argillite'   range pos-y 13.5,19.6
block zone group 'anhydrite'   range pos-y -8.8,-8.4
block zone group 'polyhalite'  range pos-y -16.1,-8.8
;
; halite properties
block prop mat 1 d 2300 b 20e9 sh 13e9
;
block zone cmodel assign wipp ...
     bulk=20.7e9 shear=12.4e9 dens=2300 act-en=12000 ...
     con-a=4.56 con-b=127 creep-rate-critical=5.39e-8 ...
     con-gas=1.987 temp = 300 ...
     exp=4.9 co-d=5.79e-36 range group 'halite'
;
; --- Anhydrite material properties ---
;
block zone cmodel assign mohr-c ...
     bulk=83.4e9 shear=27.8e9 dens=2300 fric=29 coh=27e6 ...
     range group 'anhydrite'
;
; --- 10% Polyhalite 90% Halite material properties ---
;
block zone cmodel assign wipp ...
     bulk=22.1e9 shear=13.2e9 dens=2300 act-en=12000 ...
```

```
      co-a=4.56 co-b=127 creep-rate-critical=5.39e-8 co-gas=1.987 ...
      temp=300 exp=4.9 con-d=5.21e-36 range group 'polyhalite'
;
; --- Argillaceous Halite material properties ---
block zone cmodel assign wipp ...
      bulk=20.7e9 shear=12.4e9 dens=2300 act-en=12000 ...
      con-a=4.56 con-b=127 creep-rate-critical=5.39e-8 con-gas=1.987 ...
      temp=300 exp=4.9 con-d=1.74e-35 range group 'argillite'
;
;
; clay layers
;
block contact prop mat 1 stiffness-normal 1e10 stiffness-shear .5e10 ...
  friction 5 cohesion 0
;
; boundaries
;
block edge apply stress -12.771e6,0,-12.771e6   ...
  range pos-x -1 21 pos-y 79.9 80.1
bl grid app vel-y 0 range pos-x -1,21      pos-y -81,-79.9
bl grid app vel-x 0 range pos-x -0.1,.01   pos-y -81,81
bl grid app vel-x 0 range pos-x 19.9,20.01 pos-y -81,81
block mechanical grav 0 -9.81
block insitu stress -14.576e6,0,-14.576e6 gradient-y 22563,0,22563 ...
 szz -14.576e6
block zone history stress-yy 0 0
block solve
model save 'bed1.sav'

block del range pos-x 0 5 pos-y -6 -2
block solve
model sav 'bed2.sav'

block creep timestep upper-bound 1e4
block creep timestep lower-bound 1e6
block creep timestep minimum 108
block creep timestep maximum 21600
block creep timestep lower-mult 1.2 upper-mult 0.5
block creep timestep auto
;
; FISH function to define displacement variables
;
fish def vclo
  igpt1 = bl.gp.near(0,-2.2)
  igpt2 = bl.gp.near(2.5,-2.2)
  igpm  = bl.gp.near(5.0,-4.2)
```

```
  igpb1 = bl.gp.near(0,-6.2)
  igpb2 = bl.gp.near(2.5,-6.2)
  vclo  = bl.gp.disp.y(igpb1) - bl.gp.disp.y(igpt1)
  vcloq = bl.gp.disp.y(igpb2) - bl.gp.disp.y(igpt2)
  hclo  = -2.0*bl.gp.disp.x(igpm)
  years = bl.cr.time.total/(3600*24*365.25)
end
;
; Histories
;
hist interval 200
fish history @vclo
fish history @hclo
fish history @vcloq
block creep history time
fish history @years
;
; start creeping
;
model title '5 Days after initial excavation'
block solve age 4.32e5    ; solve to 5 days
model sav ''bed_005d.sav'

model title '10 Days after initial excavation'
block solve age 8.64e5    ; solve to 10 days
model sav 'bed_010d.sav'

model title '50 Days after initial excavation'
block solve age 4.32e6    ; solve to 50 days
model sav 'bed_050d.sav'

model title '100 Days after initial excavation'
block solve age 8.64e6    ; solve to 100 days
model sav 'bed_100d.sav'

model title '200 Days after initial excavation'
block solve age 1.728e7    ; solve to 200 days
model sav 'bed_200d.sav'

model title '1 year after initial excavation'
block solve age 3.15576e7    ; solve to 1 year
model sav 'bed_1_year.sav'

ret
```

1.6.10 Compression Tests with the Crushed-Salt Model

Results of hydrostatic and shear compression tests are presented to validate the crushed-salt model in *UDEC*. Both tests involve one zone at an initially elastic state of equilibrium. The zone is allowed to creep for a period of 23 days, during which time the fractional density is monitored and compared to the analytical calculation.

1.6.10.1 Hydrostatic Compression

In the hydrostatic compression test, the confining pressure is held constant. From the incremental volumetric stress-strain law, $\Delta\sigma = K\Delta\epsilon_v^e$, and Eqs. (1.32), (1.119) and (1.122), it follows that total and creep compaction rates must also be equivalent, and we may write (using Eqs. (1.114) and (1.120))

$$\dot{\rho} = B_0\left[e^{-B_1\sigma} - 1\right]e^{B_2\rho} \tag{1.146}$$

Note that $B_0\left[e^{-B_1\sigma} - 1\right]$ is constant for this problem.

Using Eq. (1.115) for the fractional density, and given that $\rho = \rho_\circ$ at $t = 0$, integration of Eq. (1.146) yields

$$F_d = -\frac{1}{B_2\rho_f}ln\left[-B_2B_0\left[e^{-B_1\sigma} - 1\right]t + e^{-B_2\rho_\circ}\right] \tag{1.147}$$

where ρ_f is the density of intact salt.

Also, integrating Eq. (1.114), we obtain, for zero initial strain,

$$\epsilon_v = -ln\frac{\rho}{\rho_\circ} \tag{1.148}$$

The data file for this test is listed in Example 1.14. The parameters used for the automatic adjustment of the creep timestep were selected to produce the best fit within a reasonable computation time.

Numerical results for fractional density versus creep time are compared to the analytical calculations in Figure 1.31 for the four stress levels. The initial density is 1350 kg/m^3, and the final density is 2300 kg/m^3. The error on fractional density is less than 0.1% for these cases.

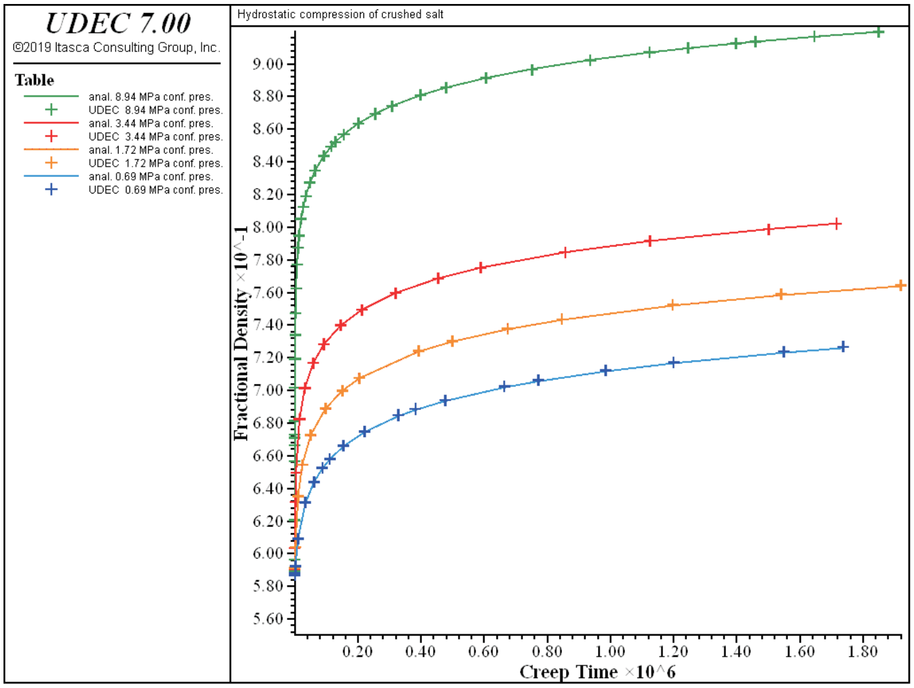

Figure 1.31 Numerical and analytical predictions for fractional density

Figure 1.31 is created by writing the analytical and *UDEC* histories to tables, and saving the tables in separate files for each stress level case. The *FISH* function **`log_it`** creates the files using *FISH* I/O routines (see Section 2.6 in the ***FISH* volume**). The table files are then input and plotted collectively in the figure.

Example 1.14 Hydrostatic compression test using the crushed-salt model

```
model new
;File:creep_14.dat
model title 'Hydrostatic compression of crushed salt'
; --- constants ---
fish def cons
  valb0   = 1.3e8
  valb1   = 0.82e-6
  valb2   = -1.72e-2
  valrho = 1350.
  valrhof= 2300.
  valbf   = 58.6383e9
  valsf   = 35.3163e9
  valcons = math.exp(-valb2*valrho)
  timeo = 0.0
end
@cons
```

```
;
fish def log_it
  array p_val (1500)
  narr = 0
  n_num = table.size(tabin)
  loop ii (1,n_num)
    tabi = tabin
    xval = table.x(tabi,ii)
    yval = table.y(tabi,ii)
    if xval > 0.0 then
      narr = narr + 1
      p_val(narr) = 'table '+string(tabi)+' add ' ...
        +string(xval)+' '+string(yval)
    endif
  endloop
  stat = file.open(filename,1,1)
  _beforesave = 2
  stat = file.write(p_val,narr)
  _aftersave = 3
  stat = file.close
end
; --- geometry ---
block config axi creep
block largestrain off
block tolerance corner-round-length 5E-3
block tolerance minimum-edge-length 1E-2
block create polygon 0,0 0,5 5,5 5,0
block zone gen quad 10.0
block zone group 'cw'
block zone cmodel assign wipp-salt bulk 1.186E8 shear 7.14E7 ...
  young 1.784E8 poisson 0.2493 activation-energy 1.2E4 ...
  constant-a 4.56 constant-b 127 constant-d 5.79E-36 ...
  creep-rate-critical 5.39E-8  con-gas 1.987  exp 4.9 temp 300 ...
  density=@valrho bulk-f=@valbf shear-f=@valsf ...
  density-f=@valrhof comp-0=@valb0 comp-1=@valb1 ...
  comp-2=@valb2 density-salt=@valrho range group 'cw'
;
fish def _point
  _zi = bl.zone.near(1,0.5)
  _gi = bl.gp.near(5,5)
end
@_point
;
fish def c_frd
  c_frd = bl.zone.prop(_zi,'density-fractional')
  c_rbulk = bl.zone.prop(_zi,'bulk')/valbf
```

```
;  in small strain:
  c_ev    = 2.*bl.gp.disp.x(_gi)/5.+bl.gp.disp.y(_gi)/5.
end
;
fish def c_frdsol
  val = - math.ln(-valb2 * valterm * ...
    bl.cr.time.total + valcons) / valb2
  c_err = 100. * (val - bl.zone.prop(_zi,'rho'))/valrhof
  c_frdsol = val/valrhof
  c_evsol  = math.ln(valrho/val)
end
;
fish def c_dt
  c_dt  = bl.cr.timestep
end
; --- histories ---
hist interval 250
block creep history time-total
fish history @c_dt
fish history @c_frd
fish history @c_frdsol
fish history @c_err
fish history @c_ev
fish history @c_evsol
fish history @c_rbulk
block gridpoint history disp-x  5,0
block gridpoint history disp-x  5,5
block gridpoint history disp-y  0,5
block gridpoint history disp-y  5,5
;
block creep timestep minimum 5.e-7 maximum 5e4 ...
  lower-bound 4000 upper-bound 5000 start 5e-7
block creep timestep auto
;
fish def val_sig
  valterm = valb0 * (math.exp(-valb1*valsig)-1.)
end
fish def _historyvs
  _rows = table.size(_tablex)
  loop _i (1, _rows)
    table.x(_tableOut,_i) = table.y(_tableX, _i)
    table.y(_tableOut,_i) = table.y(_tableY, _i)
  endloop
end
model save 'base.sav'
```

```
;
model restore 'base.sav'
; ----------- case 1 ----------------------
fish set @valsig = -8.94e6
@val_sig
;
block edge apply stress @valsig,0.0,0.0 range pos-x 4.9,5.1
block edge apply stress 0.0,0.0,@valsig range pos-y 4.9 5.1
block gridpoint apply velocity-x 0 range pos-x -.1 .1
block gridpoint apply velocity-y 0 range pos-y -.1 .1
; --- initial conditions ---
block insitu stress @valsig,0.0,@valsig szz @valsig

block solve age 1.0
block solve age 1e2
block solve age 1e3
block solve age 1e4
block solve age 1e5
block solve age 1e6
block solve age 2e6
;
hist export 1 table 1
hist export 3 table 3
hist export 4 table 4
fish set @_tableX 1
fish set @_tableY 3
fish set @_tableOut 13
@_historyvs
fish set @_tableY 4
fish set @_tableOut 14
@_historyvs
fish set @filename 'case1_3.log'
fish set @tabin 13
@log_it
fish set @filename 'case1_4.log'
fish set @tabin 14
@log_it
model save 'case1.sav'


model restore 'base.sav'
;Branch 0:case2.sav'

; ----------- case 2 ----------------------
fish set @valsig = -3.44e6
@val_sig
```

```
;
block edge apply stress @valsig,0.0,0.0 range pos-x 4.9,5.1
block edge apply stress 0.0,0.0,@valsig range pos-y 4.9 5.1
block gridpoint apply velocity-x 0 range pos-x -.1 .1
block gridpoint apply velocity-y 0 range pos-y -.1 .1
; --- initial conditions ---
block insitu stress @valsig,0.0,@valsig szz @valsig
block solve age 1
block solve age 1e2
block solve age 1e3
block solve age 1e4
block solve age 1e5
block solve age 1e6
block solve age 2e6
;
hist export 1 table 1
hist export 3 table 3
hist export 4 table 4
fish set @_tableX 1
fish set @_tableY 3
fish set @_tableOut 23
@_historyvs
fish set @_tableY 4
fish set @_tableOut 24
@_historyvs
fish set @filename 'case2_3.log'
fish set @tabin 23
@log_it
fish set @filename 'case2_4.log'
fish set @tabin 24
@log_it
model save 'case2.sav'


model restore 'base.sav'
;Branch 0:case3.sav'

; ------------ case 3 -----------------------
fish set @valsig = -1.72e6
@val_sig
;
block edge apply stress @valsig,0.0,0.0 range pos-x 4.9,5.1
block edge apply stress 0.0,0.0,@valsig range pos-y 4.9 5.1
block gridpoint apply velocity-x 0 range pos-x -.1 .1
block gridpoint apply velocity-y 0 range pos-y -.1 .1
; --- initial conditions ---
```

```
block insitu stress @valsig,0.0,@valsig szz @valsig
block solve age 1
block solve age 1e2
block solve age 1e3
block solve age 1e4
block solve age 1e5
block solve age 1e6
block solve age 2e6
;
hist export 1 table 1
hist export 3 table 3
hist export 4 table 4
fish set @_tableX 1
fish set @_tableY 3
fish set @_tableOut 33
@_historyvs
fish set @_tableY 4
fish set @_tableOut 34
@_historyvs
fish set @filename 'case3_3.log'
fish set @tabin 33
@log_it
fish set @filename 'case3_4.log'
fish set @tabin 34
@log_it
model save 'case3.sav'


model restore 'base.sav'
;Branch 0:case4.sav'

; ------------ case 4 -----------------------
fish set @valsig = -0.69e6
@val_sig
;
block edge apply stress @valsig,0.0,0.0 range pos-x 4.9,5.1
block edge apply stress 0.0,0.0,@valsig range pos-y 4.9 5.1
block gridpoint apply velocity-x 0 range pos-x -.1 .1
block gridpoint apply velocity-y 0 range pos-y -.1 .1
; --- initial conditions ---
block insitu stress @valsig,0.0,@valsig szz @valsig
block solve age 1
block solve age 1e2
block solve age 1e3
block solve age 1e4
block solve age 1e5
```

```
block solve age 1e6
block solve age 2e6
;
hist export 1 table 1
hist export 3 table 3
hist export 4 table 4
fish set @_tableX 1
fish set @_tableY 3
fish set @_tableOut 43
@_historyvs
fish set @_tableY 4
fish set @_tableOut 44
@_historyvs
fish set @filename 'case4_3.log'
fish set @tabin 43
@log_it
fish set @filename 'case4_4.log'
fish set @tabin 44
@log_it
model save 'case4.sav'


model new
call 'case1_3.log'
call 'case1_4.log'
;
call 'case2_3.log'
call 'case2_4.log'
;
call 'case3_3.log'
call 'case3_4.log'
;
call 'case4_3.log'
call 'case4_4.log'
table 13 label 'UDEC  8.94 MPa conf. pres.'
table 14 label 'anal. 8.94 MPa conf. pres.'
table 23 label 'UDEC  3.44 MPa conf. pres.'
table 24 label 'anal. 3.44 MPa conf. pres.'
table 33 label 'UDEC  1.72 MPa conf. pres.'
table 34 label 'anal. 1.72 MPa conf. pres.'
table 43 label 'UDEC  0.69 MPa conf. pres.'
table 44 label 'anal. 0.69 MPa conf. pres.'
model save 'result_14.sav'

RET
```

1.6.10.2 Shear Compression

In the shear compression test, both axial and confining stresses are kept constant. Using a notation convention in which σ_1 refers to the most negative (major) compressive stress, and σ_3 refers to the least negative (minor) compressive stress (which is also the confining stress), then the stress invariants are

$$\sigma = \frac{1}{3}(\sigma_1 + 2\sigma_3) \tag{1.149}$$

and

$$\bar{\sigma} = \sigma_3 - \sigma_1 \tag{1.150}$$

because

$$\sigma^d_{zz} = -\frac{2}{3}(\sigma_3 - \sigma_1) \tag{1.151}$$

and

$$\sigma^d_{xx} = \sigma^d_{yy} = \frac{1}{3}(\sigma_3 - \sigma_1) \tag{1.152}$$

For constant applied stresses, and neglecting the creep component, the total strain rate equals the compaction strain rate, and we may write, using $\beta = 1$ in Eq. (1.122),

$$\dot{\epsilon}_{ij} = \dot{\epsilon}^c_v \left[\frac{\delta_{ij}}{3} - \frac{\sigma^d_{ij}}{\bar{\sigma}} \right] \tag{1.153}$$

where $\dot{\epsilon}^c_v$ is given by Eq. (1.121).

Substitution of Eqs. (1.149) to (1.152) in Eq. (1.153) yields

$$\dot{\epsilon_{zz}} = \dot{\epsilon^d_v} \tag{1.154}$$

$$\dot{\epsilon_{xx}} = \dot{\epsilon_{yy}} = 0 \tag{1.155}$$

and no lateral compaction is predicted in this test for the constraints of the model.

The analytical expression for the fractional density is similar to that obtained for the hydrostatic compression test (i.e., Eq. (1.147)).

Two tests are performed: an unconfined compression test and a biaxial compression test. In both tests, $\sigma_1 = -8.97$ MPa. $\sigma_3 = 0.0$ in the unconfined compression test, and $\sigma_3 = -6.90$ MPa in the biaxial compression test. Initial and final density, and shear and bulk moduli, are identical to those adopted in the hydrostatic compression test. Example 1.15 contains the data file for these tests.

Example 1.15 Unconfined and biaxial compression tests using the crushed-salt model

```
model new
;File:creep_15.dat
model title 'compression testing of crushed salt'
; --- constants ---
fish def cons
  valsig1 = -8.97e6
  valsig = (valsig1 + 2.0 * valsig3) / 3.0
  valb0  = 1.3e8
  valb1  = 0.82e-6
  valb2  = -1.72e-2
  valrho = 1350.
  valrhof= 2300.
  valbf  = 58.6383e9
  valsf  = 35.3163e9
  valterm = valb0 * (math.exp(-valb1*valsig)-1.)
  valcons = math.exp(-valb2*valrho)
end
fish set @valsig3 = 0.0 ; uniaxial    consolidation
@cons
; --- geometry ---
block config axi creep
block largestrain off
block tolerance corner-round-length 5E-3
block tolerance minimum-edge-length 1E-2
block create polygon 0,0 0,5 5,5 5,0
block zone gen quad 10.0
;block zone gen edge 10
block zone group 'cw'
block zone cmodel assign wipp-salt bulk 1.186E8 shear 7.14E7 ...
  activation-energy 1.2E4 constant-a 4.56 constant-b 127 ...
  constant-d 5.79E-36 ...
  creep-rate-critical 5.39E-8 con-gas 1.987 exp 4.9 temperature 300 ...
  density=@valrho bulk-f=@valbf sh-f=@valsf dens-f=@valrhof ...
  comp-0=@valb0 comp-1=@valb1 ...
```

```
  comp-2=@valb2 density-salt=@valrho range group 'cw'
; --- fish function ---
fish def _point
  _zi = bl.zone.near(1,0.5)
  _gi = bl.gp.near(5,5)
end
@_point
fish def c_frd
  c_frd = bl.zone.prop(_zi,'frac_d')
  c_rbulk = bl.zone.prop(_zi,'bulk_mod')/valbf
;  in small strain:
  c_ev    = 2.*bl.gp.disp.x(_gi)/5.+bl.gp.disp.y(_gi)/5.
end
fish def c_frdsol
  val = - math.ln(-valb2 * valterm * ...
    bl.cr.time.total + valcons) / valb2
  c_err = 100. * (val - bl.zone.prop(_zi,'rho'))/valrhof
  c_frdsol = val/valrhof
  c_evsol  = math.ln(valrho/val)
end
fish def c_dt
  c_dt = crtdel
end
; --- histories ---
hist interval 250
block creep history time-total
fish history @c_dt
fish history @c_frd
fish history @c_frdsol
fish history @c_err
fish history @c_ev
fish history @c_evsol
fish history @c_rbulk
block gridpoint history disp-x  5,0
block gridpoint history disp-x  5,5
block gridpoint history disp-y  0,5
block gridpoint history disp-y  5,5
; --- test ---
block creep timestep minimum 5.e-7 max 5e4 lower-bound 4000 ...
  upper-bound 5000 latency 1 lower-mul 1.01 upper-mul 0.99
block creep timestep auto
;
model save 'common_15.sav'

model restore 'common_15.sav'
```

```
; --- boundary conditions ---
block edge apply stress @valsig3,0.0,0.0 range pos-x 4.9,5.1
block edge apply stress 0.0,0.0,@valsig1 range pos-y 4.9,5.1
block insitu stress @valsig3,0.0,@valsig1 szz @valsig3
block gridpoint apply velocity-x 0 range pos-x -0.1,0.1
block gridpoint apply velocity-y 0 range pos-y -0.1,0.1

block solve age 1
block solve age 1e2
block solve age 1e3
block solve age 1e4
block solve age 1e5
block solve age 1e6
block solve age 2e6

model save 'Uniax_15.sav'

model restore 'common_15.sav'

; shear    consolidation
fish set @valsig3 -6.90e6
@cons
; --- boundary conditions ---
block edge apply stress @valsig3,0.0,0.0 range pos-x 4.9,5.1
block edge apply stress 0.0,0.0,@valsig1 range pos-y 4.9,5.1
block insitu stress @valsig3,0.0,@valsig1 szz @valsig3
block gridpoint apply velocity-x 0 range pos-x -0.1,0.1
block gridpoint apply velocity-y 0 range pos-y -0.1,0.1

block solve age 1
block solve age 1e2
block solve age 1e3
block solve age 1e4
block solve age 1e5
block solve age 1e6
block solve age 2e6

model save 'shear_15.sav'

ret
```

Numerical values of fractional density are compared to analytical values versus creep time for the unconfined compression test in Figure 1.32, and for the biaxial compression test in Figure 1.33. The error in fractional density is less than 0.1%. The histories of axial and lateral displacements for the unconfined compression test are shown in Figure 1.34, and for the biaxial compression test in Figure 1.35. As expected, no lateral displacement is calculated in these tests.

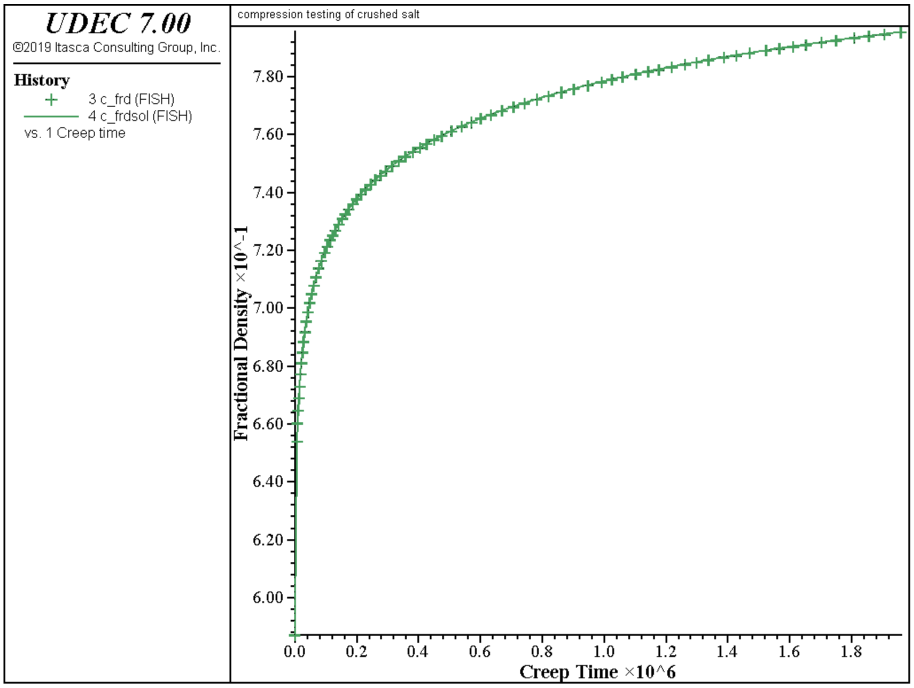

Figure 1.32 Numerical and analytical predictions for fractional density for uniaxial compression

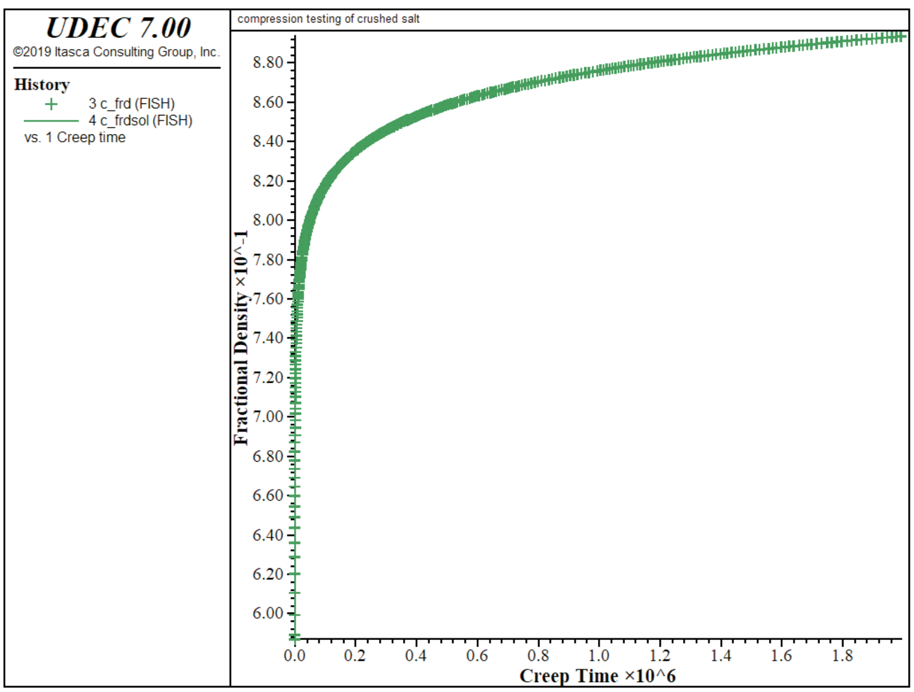

Figure 1.33 Numerical and analytical predictions for fractional density for biaxial compression

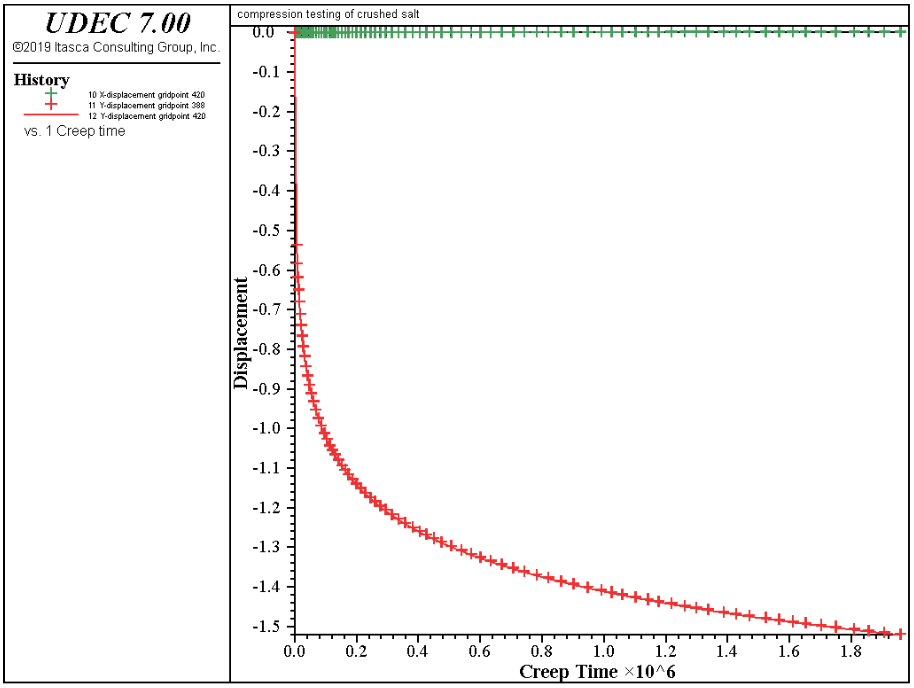

Figure 1.34 Histories of axial and lateral displacement for uniaxial compression

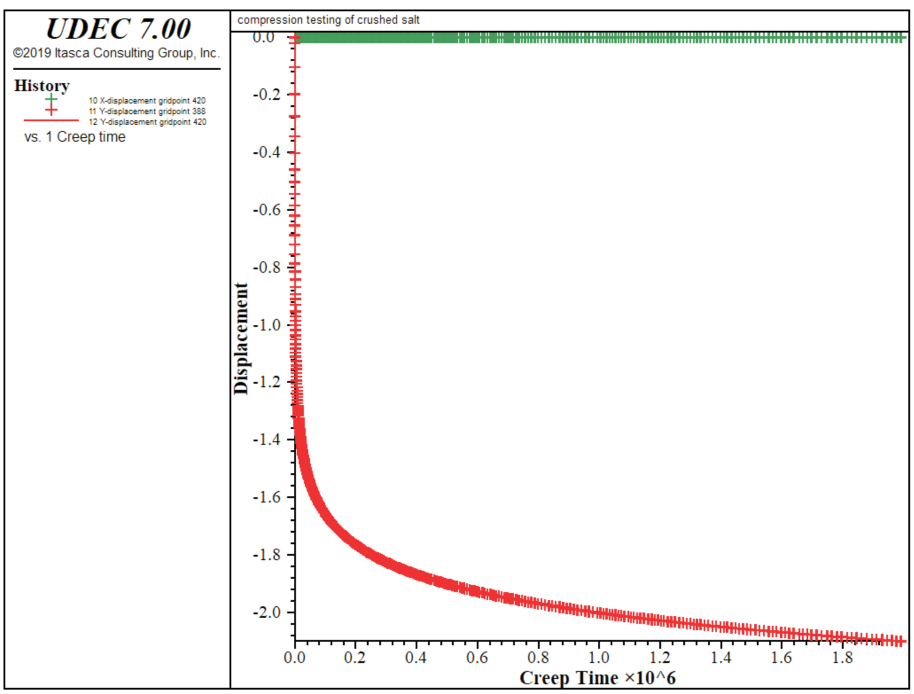

Figure 1.35 Histories of axial and lateral displacement for biaxial compression

1.6.11 Loaded Block – Burgers Model

Example 1.16 corresponds to a square block of material loaded at the top surface with a constant stress. The block is composed of material that obeys the Burgers model, in which all elements are “active” (i.e., both moduli have similar values, and both viscosities have similar values). In this case, we expect both Kelvin and Maxwell components to influence the response.

The test was performed with the Burgers viscoelastic model (**burgers**). (See Section 1.2.5.)

Section 1.5.6 shows the displacement response of the top of the block versus creep time. There is an instantaneous elastic response caused by the Maxwell spring, followed by continuous creep in the long-term caused by the Maxwell viscosity. The curvature in the short-term is due to the relaxation of the Kelvin component.

Example 1.16 Loaded block composed of the Burgers model material

```
model new
;file: creep_16.dat
model title 'tests of Burgers model'
block config creep
block tolerance corner-round-length .001
block create polygon 0 0 0 5 5 5 5 0
block zone gen quad 1
block zone cmodel assign burgers
block edge apply stress 0,0,-.1 range pos-y 4.9 5.1
block gridpoint apply vel-y 0 range pos-y -.1 .1
block gridpoint history disp-y  2,5
block creep history time-total
model save 'common_16.sav'

;
model restore 'common_16.sav'

model title 'test of Burgers viscoelastic model'
block zone property dens 1 bulk 2 shear-kelvin 1 shear-maxwell 1 ...
  visc-k 1 visc-m 3
block step 500
block grid reset vel
block creep timestep fix  .001
block step 200
block creep timestep fix  .005
block step 800
model sav 'creep_16a.sav'


;
```

```
model restore 'common_16.sav'

model title 'test of Maxwell viscoelastic model'
block zone property dens 1 bulk 2 shear-k 1 shear-m 1 ...
  visc-k 1000 visc-m 3
block step 500
block grid reset vel
block creep timestep fix  .001
block step 200
block creep timestep fix  .005
block step 800
model save 'creep_16b.sav'


;
model restore 'common_16.sav'

model title 'test of Burgers viscoelastic model'
block zone property dens 1 bulk 2 shear-k 1 shear-m 50 ...
  visc-k 1 visc-m 1e10
block step 500
block  grid reset vel
block creep timestep fix  .001
block step 200
block creep timestep fix  .005
block step 800
model save 'creep_16c.sav'

ret
```

If the **property** command in Example 1.16 is replaced by the line

```
block zone prop dens 1 bulk 2 shear-kelvin 1 shear-maxwell 1 ...
  visc-kelvin 1000 visc-maxwell 3
```

the response resembles that of a Maxwell model alone, since the Kelvin section of the Burgers model is made almost rigid by the use of a large viscosity (compared to the Maxwell viscosity). Figure 1.37 shows the response. This is similar to the response that would be obtained with model **maxwell**, using similar properties: a shear modulus of 1 unit and a viscosity of 3 units.

If the **property** command in Example 1.16 is replaced by the line

```
block zone prop dens 1 bulk 2 shear-k 1 shear-m 50 visc-k 1 visc-m 1e10
```

the response resembles that of a Kelvin model alone, since the Maxwell section of the Burgers model is made almost rigid by the use of a large viscosity and a large modulus. However, the modulus **`shear-m`** is not made arbitrarily large because the "static" convergence would be poor.

Figure 1.38 shows the response: the displacement history exhibits almost no initial jump (since the elastic modulus of the Maxwell section is high), and there is no long-term creep, which is characteristic of the Kelvin model.

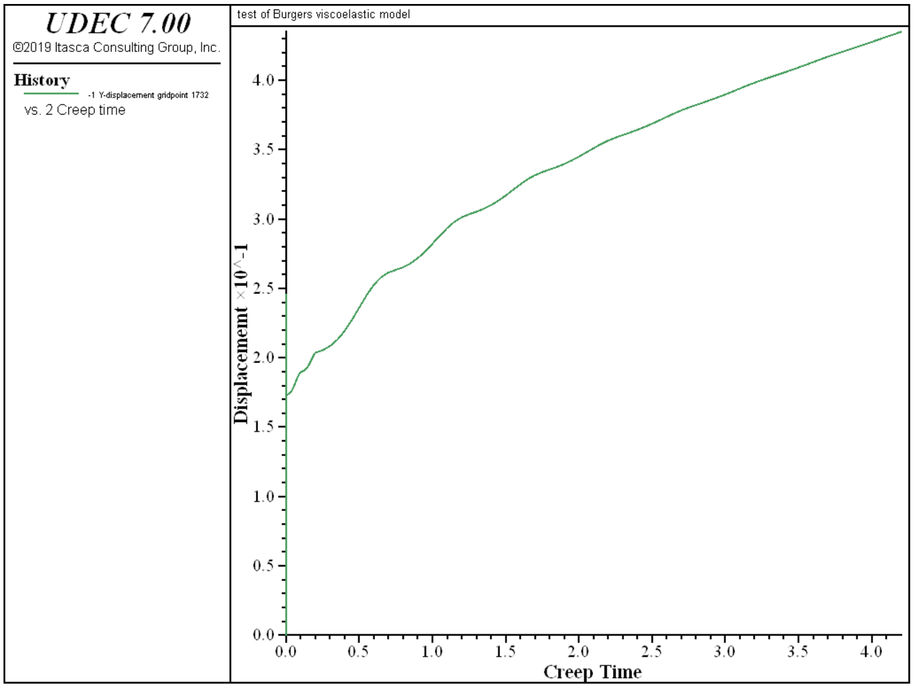

Figure 1.36 Vertical displacement versus time, for the Burgers model

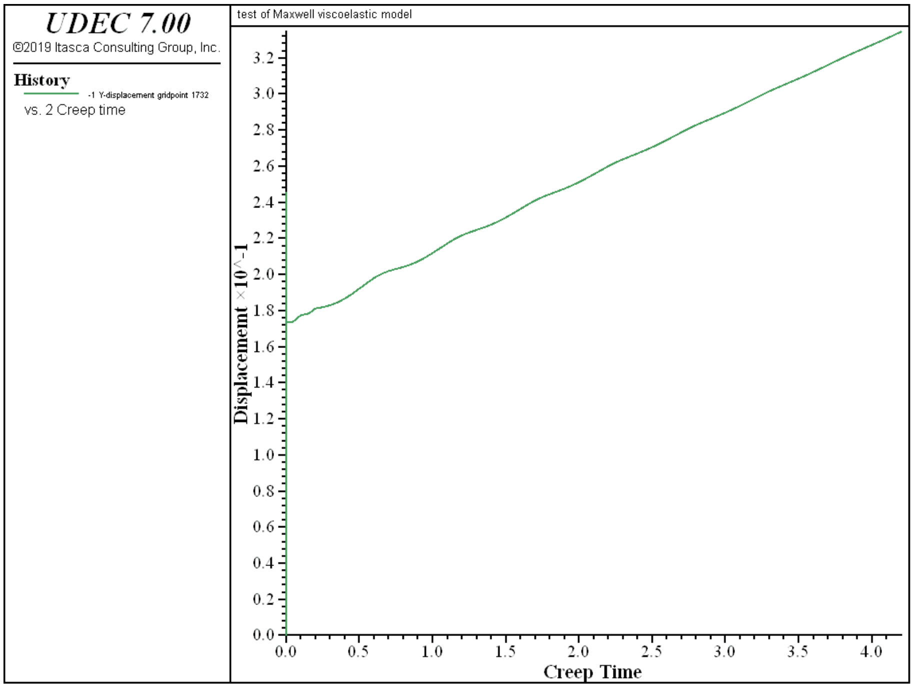

Figure 1.37 Vertical displacement versus time, for Maxwell section only active

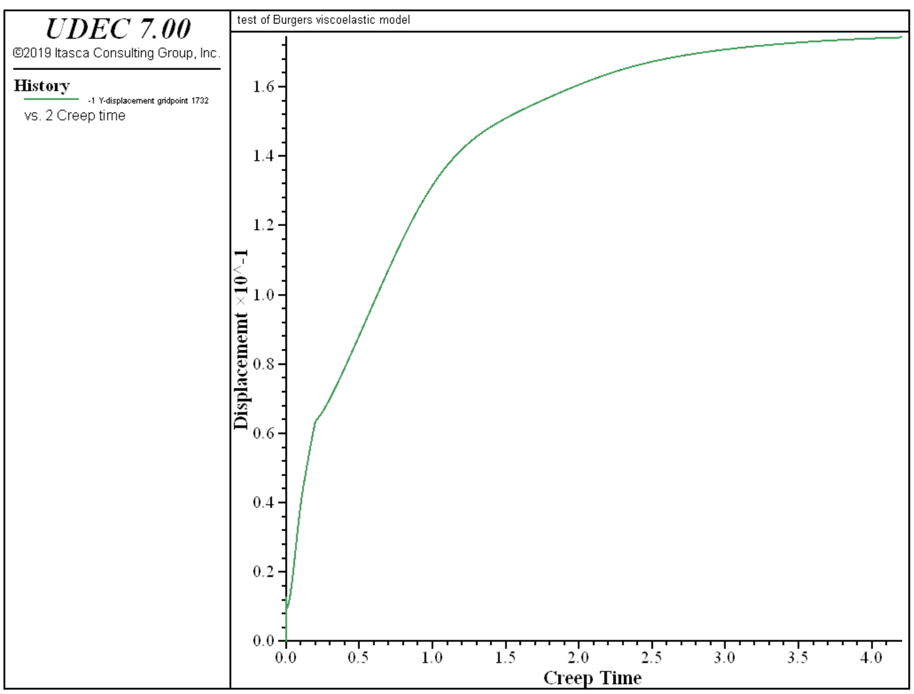

Figure 1.38 Vertical displacement versus time, for Kelvin section only active

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