2 Sliding Block between Two Slightly Skewed Rigid Walls

2.1 Problem Statement

This problem involves the sliding of an elastic block between two nearly parallel walls (see Figure 2.1), and is derived from a similar problem in Wart et al. (1984)*. A pressure is applied to one edge of the block such that the block moves, the initial gap is closed, and the normal stress on the contact faces between the block and the walls increases. The increased normal stress causes an increase in the shear resistance through friction on the surface, and the block stops. The problem involves computation of the block displacement parallel to the direction of applied pressure.

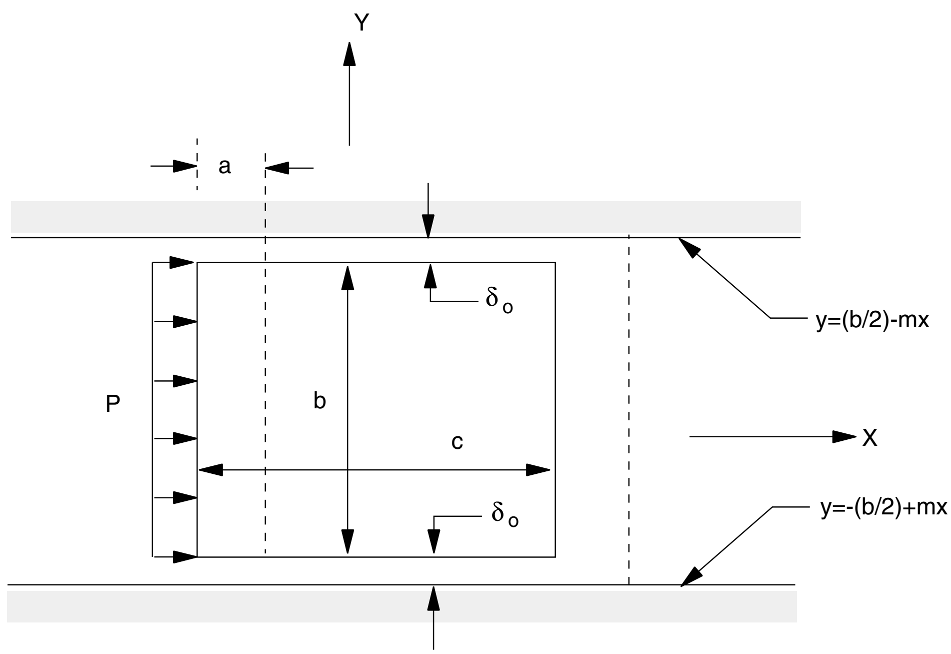

Figure 2.1 Sliding block between two slightly skewed rigid walls

The objective of this problem is to demonstrate

(a) the computation of correct stresses and displacements for a problem involving nonlinear geometry and constitutive relations; and

(b) the ability of *UDEC* to handle relatively large displacements.

The problem is solved for several input values:

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<u>Geometry</u>	
block height	b = 1.0 m
block length	c = 1.0 m
skew slope	$m = 1.0 \times 10^{-3}$
initial gap for both surfaces	$\delta_0 = 1.0 \times 10^{-5}$ m
<u>Material Properties</u>	
modulus of elasticity of the block	E = 20,000 MPa
Poisson's ratio of the block	ν = 0.25
friction angle of the sliding surfaces	ϕ = 30°
joint normal stiffness	k_n = 80,000 MPa / m
<u>Loads</u>	
pressure	P = 0.5, 1.0 and 2.0 MPa

2.2 Analytical Solution

The analytical solution is based on simple geometrical and stress-strain relations. As the block slides due to imposed pressure, P, the gap will close, and the normal stress across the joint will increase due to increased confinement given by the skewed walls.

The assumptions related to the analytical solution include

(1) the block has a linear stress-strain relation;

(2) the skew angle is small (therefore, all of the resistance to sliding is by friction), and the pressure on the block is uniform over the top and bottom surfaces; and

(3) the walls are incompressible relative to the block.

The sliding distance can be divided into two parts:

$$a = a_\delta + a_\sigma \tag{2.1}$$

where a = distance of sliding to reach equilibrium;
a_δ = distance of sliding until the initial gap is closed; and
a_σ = distance of sliding as the normal stress increases.

The distance of sliding before gap closure is

$$a_\delta = \frac{\delta_0}{m} \tag{2.2}$$

where δ_0 = the initial gap at both the top and bottom of the block; and

m = slope of the skewed walls (see Figure 2.1).

The block will stop sliding when the frictional resistance equals the applied load. The shear stress due to friction is given by

$$|\tau_\mathrm{f}| = -\mu\sigma = -\sigma \tan\phi \tag{2.3}$$

where τ_f = shear stress due to friction;
σ = normal stress across the joint;
μ = coefficient of friction of the joint; and
ϕ = friction angle of the joint.

Using the stress-strain and geometric relations between the sliding distance and strain, the normal stress across each joint is given by

$$\sigma = \frac{2m\, a_\sigma}{b}\, E^* \tag{2.4}$$

where b = block height; and
E^*= equivalent elasticity of the block joint system.

The equivalent elasticity of the block joint system is given by

$$\frac{1}{E^*} = \frac{1}{E} + \frac{2}{k_n \cdot b} \tag{2.5}$$

where E = modulus of elasticity for the block; and
k_n = joint normal stiffness.

The friction forces on each sliding surface can then be found by substituting Eq. (2.4) into Eq. (2.3). Summing the forces in the x-direction and rearranging terms gives

$$a_\sigma = \frac{P b^2}{4c\; m\; E^*\; \tan\phi} \tag{2.6}$$

where c = length of each sliding surface.

The preceding solution is for plane stress conditions. The solution for plane strain conditions can be found by substituting $E/(1-\nu^2)$ for E in Eq. (2.5).

2.3 *UDEC* Model

Because of the symmetry about the y = 0 line, only the upper half of the problem is studied. The elastic block is discretized into constant strain finite-difference triangles (see Figure 2.2). The problem is run using maximum zone edge lengths of 0.2, 0.1 and 0.05 m. Symmetry conditions are specified by assigning a zero vertical velocity to the lower horizontal boundary. The initial gap is obtained by assigning an appropriate vertical velocity to the upper rigid block, and allowing it to move upward the specified distance. Once the upper block reaches the correct position, it is immobilized.

The *UDEC* data files to run this problem are listed in Examples 2.1, 2.2 and 2.3. "SKEW_IN.DAT" in Example 2.1 specifies the model parameters and runs a series of nine tests. "SKEW_PR.DAT" in Example 2.2 processes the *UDEC* results to calculate the average sliding distance. "SKEW.DAT" in Example 2.3 contains the *UDEC* commands to run the model simulation. The results for sliding distance are written to the file "UDEC.LOG" for comparison to the analytical solution.

Note that adaptive global damping (**block mechanical damping global**) is used for this analysis. The pressure loading is applied instantaneously, but the initial global damping is sufficient to minimize transient effects. Local damping could also be used, but the loading would need to be applied gradually to minimize the transient effects.

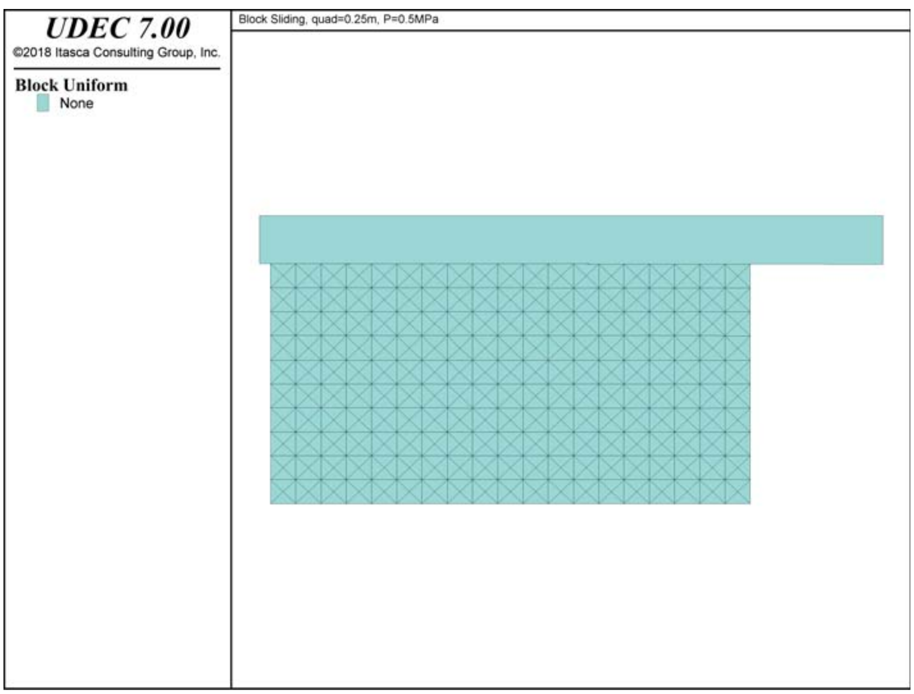

Figure 2.2 ***Position of upper rigid block, and discretization of elastic block into constant strain finite difference triangles for nominal zone size of 0.055 m***

2.4 Results and Discussion

The results for various zone size assumptions are shown in Table 2.1. These results indicate that the calculation for sliding distance is somewhat dependent on the discretization of the elastic block. The dependency is due to the nonuniform vertical stress that is present in the elastic block at equilibrium. The analytical solution assumes that the pressure on the block is uniform over the top and bottom surfaces, as given by Eq. (2.4). For P = 1.0 MPa, Eq. (2.4) indicates that the normal stress on the top and bottom surfaces would be approximately 0.87 MPa. Figures 2.3 through 2.5 show the equilibrium distribution of normal stress on the top surface of the block, and indicate that the distribution of normal stress on joint is not uniform. As the zone size is decreased, the area of uniform stress along the joint increases.

Table 2.1 Comparison of UDEC results using various zone sizes with analytical solution for sliding block between two slightly skewed rigid blocks. (Results shown are total sliding distance, m.)

Pressure (MPa)	Analytic Solution	Zone* Size	UDEC Results	Error (%)
0.5	0.0256	0.055	0.0233	−8.6
		0.125	0.0234	−6.6
		0.205	0.0260	1.6
1.0	0.0412	0.055	0.0367	−10.9
		0.125	0.0368	−10.7
		0.205	0.0407	−1.2
2.0	0.0724	0.055	0.0634	−12.4
		0.125	0.0635	−12.3
		0.205	0.0691	−4.7

*actual maximum zone edge length

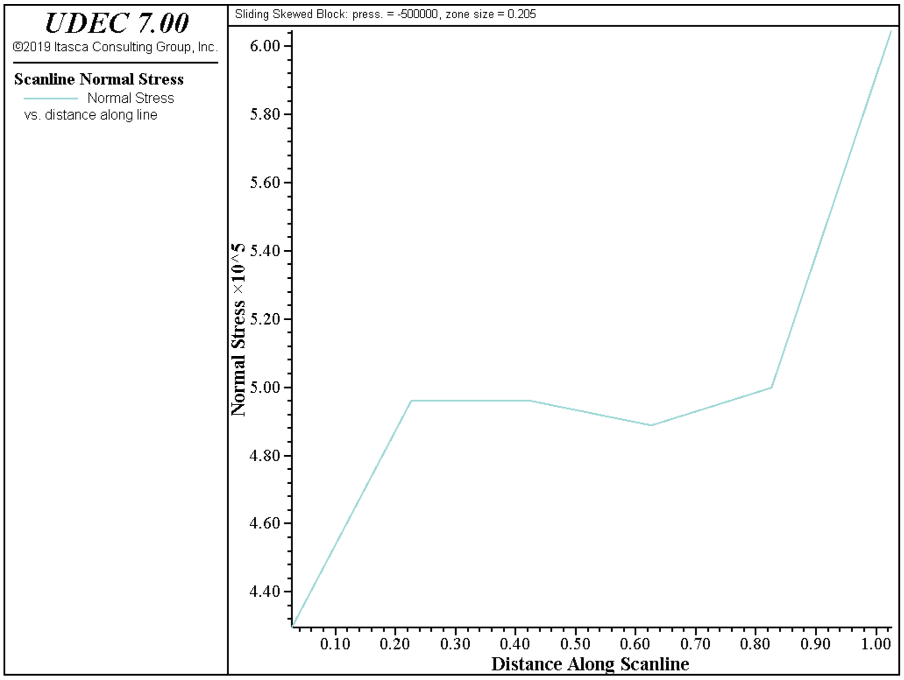

Figure 2.3 Distribution of normal stress on top surface of sliding elastic block for nominal zone size = 0.205 m

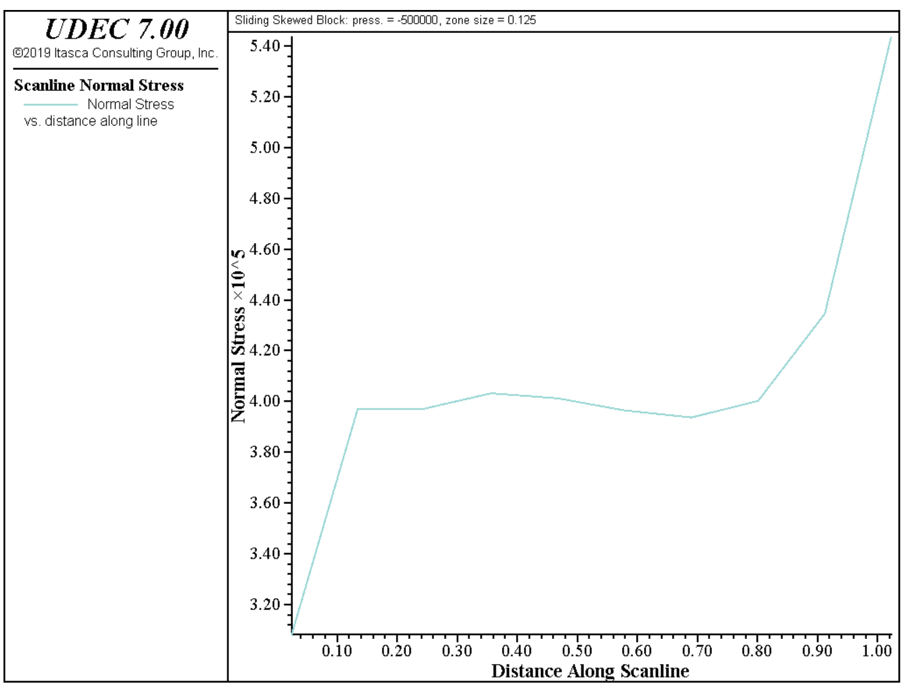

Figure 2.4 Distribution of normal stress on top surface of sliding elastic block for nominal zone size = 0.125 m

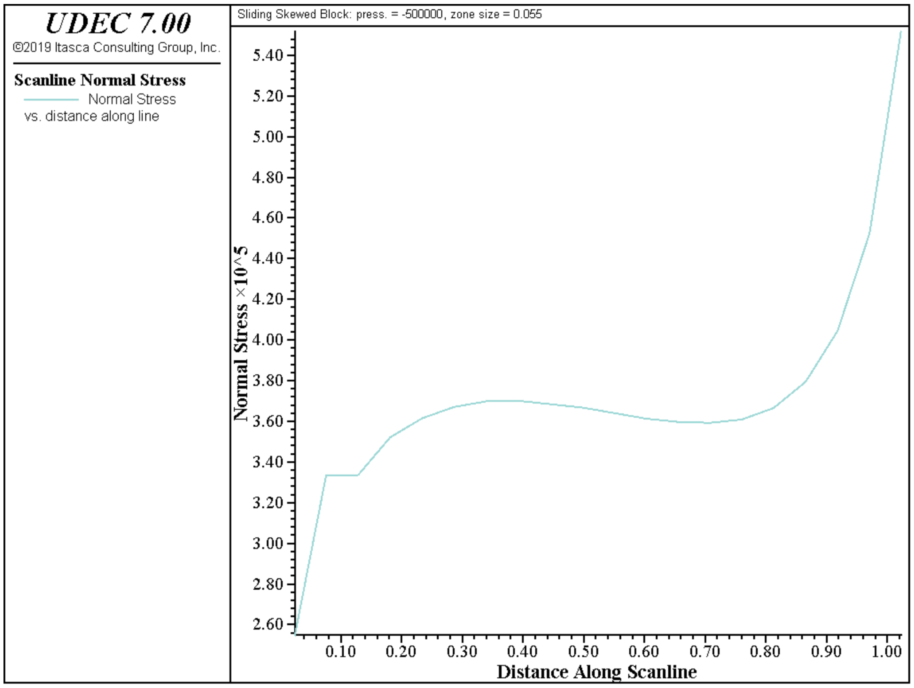

Figure 2.5 Distribution of normal stress on top surface of sliding elastic block for nominal zone size = 0.055 m

The analytical solution presented by Wart et al. (1984) does not include discussion of the influence of the joint shear stiffness parameter. The joint shear stiffness, k_s, is related to the joint shear stress, σ, by the amount of shear displacement when the surfaces are in contact – i.e.,

$$\tau = k_s \, a_\sigma \tag{2.7}$$

subject to the limitations of Eq. (2.3). In this problem, the analytical solution is therefore valid for joint shear stiffnesses greater than the minimum values necessary to achieve an equilibrium shear stress within the calculated sliding distance. This value is obtained by dividing the limiting shear stress, σ, at a specified pressure, P, by the amount of the sliding distance, a_σ, as the normal stress increases. For P = 1 MPa, τ = 0.5 MPa and a_σ = 0.0312 m, k_s must therefore be 16 MPa/m or greater.

2.5 Reference

Wart, R. J., E. L. Skiba and R. H. Curtis. “Benchmark Problems for Repository Design Model,” NUREG/CR-3636 (February 1984).

2.6 Listing of Data Files

Example 2.1 SKEW_IN.DAT

```
model new
;--------------------------------------------------------
; Verification test:
; Sliding block between two slightly skewed rigid walls
; Joint stiffnesses :stiffness-normal=stiffness-shear=8e10
;
; Input data
;--------------------------------------------------------
;
call 'skew_pr.dat'
;
; --- zone size
;
fish set @z_size = 0.205
;
; --- pressure
;
fish set @p_load = -0.5e6
;
; --- velocity to close gap
;
fish set @gap_vel = 0.77639
;
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.125
fish set @p_load = -0.5e6
fish set @gap_vel = 1.18751
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.055
fish set @p_load = -0.5e6
fish set @gap_vel = 2.70709
call 'skew.dat'
;
model new
```

```
;
call 'skew_pr.dat'
fish set @z_size = 0.205
fish set @p_load = -1.0e6
fish set @gap_vel = 0.77639
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.125
fish set @p_load = -1.0e6
fish set @gap_vel = 1.18751
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.055
fish set @p_load = -1.0e6
fish set @gap_vel = 2.70709
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.205
fish set @p_load = -2.0e6
fish set @gap_vel = 0.77639
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.125
fish set @p_load = -2.0e6
fish set @gap_vel = 1.18751
call 'skew.dat'
;
model new
;
call 'skew_pr.dat'
fish set @z_size = 0.055
fish set @p_load = -2.0e6
fish set @gap_vel = 2.70709
call 'skew.dat'
```

```
;
ret
```

Example 2.2 SKEW_PR.DAT

```
;---------------------------------------------------------
; Verification test:
; Sliding block between two slightly skewed rigid walls
; Joint stiffnesses :stiffness-normal=stiffness-shear=8e10
;
; Model preparation and post-processing
;---------------------------------------------------------
fish def setup
;
; --- model parameters ---
;
   z_s = z_size
   p_l = p_load
   g_v = gap_vel
;
; --- title of simulation ---
;
   run_t = 'Sliding Skewed Block: press. = '+string(p_l)
   run_t = run_t+', zone size = '+string(z_s)
   pa    = int(-p_l / 1e5)
   za    = int(100 * z_s)
   nam_t = 'p'+string(pa)+'_z'+string(za)+'.sav'
end
;
; --- UDEC average sliding distance ---
;
fish def slide_dist
   sum = 0.0
   nsum = 0
   ib = block.head
   loop while ib # 0
      ig = bl.gp(ib)
      loop while ig # 0
         nsum = nsum + 1
         sum = sum + bl.gp.disp.x(ig)
         ig = bl.gp.next(ig)
      endloop
      ib = bl.next(ib)
   endloop
   if nsum # 0 then
```

```
      s_dist = sum / nsum
   endif
end
ret
```

Example 2.3 SKEW.DAT

```
;
;title 'Sliding block between two slightly skewed rigid walls'
; Load P=0.5 MPa and zone size=0.205m
; Joint stiffnesses :stiffness-normal=stiffness-shear=8e10
;
@setup
block tolerance corner-round-length 0.001
; set geometry using symmetry:
block create polygon 0 0 0 .6 1.3 .6 1.3 0
block cut crack 0 .5 1.3 .4987
block cut crack 1 0 1 .5
block delete range position-x 1 1.5 position-y 0 .5
; sliding block is fully deformable:
block zone gen quad @z_s range pos-x 0 .5 pos-y 0 1
; material properties
block property material 1 density 2000 shear 8e9 bulk 1.333e10
block contact property material 1 stiffness-normal 8e10 ...
  stiffness-shear 8e10 friction 30.0
; histories:
hist interval 100
block mechanical hist solve-local-ratio
block gridpoint history disp-x .5 0
block contact history displacement-normal 1.1 0.5
block contact history displacement-shear 1.1 0.5
block contact history stress-shear 1.1 0.5
model display hist 1
;
; fix bottom (symmetry line)
bl grid app vel-y=0 range pos-x 0 1.3 pos-y -.01 .01
;
; initialization with very small insitu stress
block insitu  stress -1e-6 0 -1e-6
;
; use cyc 0 to get initial time step; dividing the 1e-5 gap by the
; initial time step will give the velocity to be applied
;
block cycle 0
;
```

```
; open the 1e-5 gap between the block and the wall:
; velocity of 2.70709 assumes time step of 3.694e-6
; velocity of 1.18751 assumes time step of 8.421e-6
; velocity of 0.77639 assumes time step of 1.288e-5
block fix all range position-y 0.5 0.7
block apply vel-y @g_v  range pos-y 0.5 0.7
block cycle 1
;
block mechanical damping global
;
; fix the wall:
block apply velocity-y 0.0 range position-y 0.5 0.7
;
; set loading
block edge apply stress @p_l 0 0 range pos-x -.1 .2 pos-y -.1 .5
;re-establish symmetry line
bl grid app velocity-y 0 range pos-x 0,1.3 pos-y -.001 .001
;
block solve ratio 1e-5
;
model title @run_t
;
model save @nam_t
;
@slide_dist
;
log on
fish list @run_t
fish list @s_dist
log off
```